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Using Auctions to Improve Tournaments:

Theory and a Study of Defense Acquisition

by

Richard Lee Fullerton, B.S., M.S.

Dissertation

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Using Auctions to Improve Tournaments:

Theory and a Study of Defense Acquisition

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Dedicated to

Brenda

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struggled to invent an elusive proof for discrete auctions. Gordon Hanson was kind enough to take time from his own research and volunteer for my committee. Finally, Doug Dacy not only challenged me in macroeconomics but he also kept me focused on my priorities in the Air Force. His friendly advice over the past three years has been so timely and astute it is almost inconceivable he ever had the poor judgment to join the Marines (twice) instead of the Air Corps.

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INTRODUCTION

Tournaments are held for a variety of reasons. In sports they are entertaining and induce athletes to put forth their best efforts while "going for the gold." In economic settings, tournaments are conducted because they are relatively simple mechanisms for mitigating the effects of moral hazard. During the 1980s, the economic theory of tournaments was directed primarily towards labor markets and the optimal use of competition to induce efficient levels of effort.[1] This emphasis was understandable given the prevalence of tournament-style policies in "up-or-out" promotion schemes. More recently, tournament theory has been revived by Taylor's work on research tournaments.[2] Though research contests were largely overlooked by the earlier tournament literature, in practice they account for enormous sums of public and private funds. For example, the 1991 Advanced Tactical Fighter (ATF) prototype "fly-off" contest sponsored by the Department of Defense (DOD) relied on head-to-head competition between two teams of firms, offering the winning team a long-term production contract estimated to be worth more than $90 billion.[3] Yet, despite the substantial theoretical work on labor and research tournaments, there are

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[1] e.g., Green and Stokey, 1983; Lazear and Rosen, 1981; McLauglin, 1988; Nalebluff and Stiglitz, 1983; O'Keefe, Viscusi and Zeckhauser, 1984; and Rosen, 1986.
[2] Curtis Taylor, 1995.
[3] Schwartz, et al. "The $93 Billion Dogfight," May 6, 1991.

still serious practical problems to implementing efficient contests stemming from the distribution of information.

Incomplete or asymmetric information almost always disrupts economic efficiency, regardless of the context of the problem. In tournaments, a lack of information about contestant types and abilities creates adverse selection problems for the sponsor who must limit entry and decide which potential contestants should be allowed to participate. Similarly, the proper size of the efficient prize is easy to compute in theory, but in practice it is difficult for the sponsor to determine the optimum prize size because it depends upon information about the effort and performance trade-offs of contestants which the sponsor almost certainly does not know. The fact that sponsors are insufficiently informed to conduct efficient tournaments is not surprising and has been acknowledged in most of the previous literature. What is surprising, however, is that economists have not attempted to incorporate into tournaments some of the familiar mechanisms that are known to ease the burden of information in other economic environments.

Auctions are easy to implement, have been used for thousands of years to buy and sell goods, and are proven to be useful for conducting economic exchanges with limited information -- yet they have been ignored as a means for restoring efficiency to tournaments. Nevertheless, auctions can be ideal mechanisms for alleviating problems of adverse selection and prize selection in tournaments with

limited information. If properly incorporated, auctions can help sponsors select the best-qualified contestants for tournament entry, and they can substantially reduce the cost of conducting a tournament by requiring less information and inducing greater effort by contestants. In short, auctions can help resolve the problems that have long been considered significant drawbacks of the tournament literature.

The first two chapters of this dissertation are theoretical, examining the use and effectiveness of auctions for improving efficiency and reducing the information burden on tournament sponsors. Throughout the theory portion of this dissertation, I use Taylor's model of a research tournament to demonstrate the proper use of auctions in a tournament environment, although one can presumably extend these findings to tournaments in all fields of study.

Since selecting the right contestants is a major problem in tournaments and the first difficulty encountered by sponsors, I address adverse selection first in chapter one. Reviewing the earlier literature on labor tournaments, Kenneth McLaughlin has acknowledged that adverse selection is, "The real problem of tournaments with heterogeneous contestants . . ." and "The inefficiencies due to adverse selection generate a demand for information."[4] I distinguish between two distinct forms of contestant heterogeneity which have two very different implications for the efficient selection of tournament contestants. The least troublesome form of

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[4] Kenneth McLaughlin, 1988, pg. 245, 248.

heterogeneity is when contestants have different costs of effort or productivity. When heterogeneity is of this form, auctions can easily be used to induce efficient self-selection by contestants. The more difficult problem of adverse selection occurs when contestants have different starting positions, giving one contestant an advantage over others prior to ever putting forth effort. With this kind of heterogeneity, simple auctions are not efficient. However, I suggest a way to transform the basic auction and prize structure to restore efficiency and ensure the best contestants are chosen by the tournament sponsor. Therefore, by judiciously incorporating auctions into tournaments, the sponsor can reduce or eliminate the ill effects of adverse selection.

In chapter two, I address another major decision sponsors must make when conducting a tournament -- the proper prize to award. As mentioned earlier, the optimal tournament requires a very specific reward to induce enough effort by contestants without the sponsor giving away more than is necessary. In real life this is a very difficult problem. How can the sponsor accurately determine how large to set the prize in order to elicit a relatively efficient outcome? In chapter two I show that the sponsor can avoid this problem entirely and induce greater effort from contestants by conducting an auction at the end of the tournament. In this auction-style tournament, contestants not only submit their best innovations at the end of the contest, but they also submit "bids" which are the prizes they demand if chosen as

the contest's winner. By creating this additional prize competition at the end of the tournament the sponsor retains all of the salient features of the standard fixed-prize tournament and simultaneously reduces the expected cost of inducing any level of contestant effort. Therefore, the sponsor gets a better tournament and does not have to predetermine the size of the winner's prize -- eliminating another information burden on the sponsor.

In chapter three I study Department of Defense acquisition policies to see how closely they conform to the basic requirements of efficient research tournaments. Even though DOD procurement is ostensibly based on competition, I show that past and current DOD practices deviate substantially from the requirements of an efficient tournament environment. In particular, DOD procurement efforts are hampered by a lack of credible commitment, improper contracting, excessive oversight, and a number of other problems predominantly self-imposed by the government, which may cause substantial purchasing inefficiencies in the military's multi-billion dollar budget.

Chapter 1

Selecting Contestants for Rank-Order Tournaments

In the economic literature on tournaments there are few practical suggestions on how to prevent the adverse selection of poor contestants. Auctions appear to be ideally suited for selecting contestants in rank-order tournaments because the best qualified contestants also value participation the most. However, simple auctions are efficient only when contestants differ in their costs of effort. In contrast, when contestants differ in starting technologies, there is no symmetric bidding equilibrium for the entry auction. Yet, the sponsor can restore efficiency at an arbitrarily small cost by tying the size of the tournament winner's prize to the size of the winner's entry bid. This chapter demonstrates how to use auctions to select the best contestants for tournaments.

1.1: INTRODUCTION

Tournaments have long been used to induce contestants to put forth effort in athletic and economic settings. While much has been written about various ways to mitigate the effects of moral hazard in tournament models, relatively little is known about useful solutions to the problem of adverse selection even though it is a fundamental drawback of all tournaments. Surveying the literature on tournaments, Kenneth McLaughlin (1988) acknowledged the problem of adverse selection in tournaments, writing, "The real problem of tournaments with heterogeneous contestants arises if the contestant's types cannot be identified. Since tournaments which are mixed ex post do not induce optimal effort and it is costly to induce self-sorting of contestants into their respective leagues, the outcome is not efficient."[1]

Adverse selection arises because the number of contestants must be limited to create efficient levels of tournament competition. Even tournaments that would appear to welcome all competitors, like the popular U.S. Open Golf Tournament, are not really "open" to all contestants.[2] Tournament entry must be limited for two primary reasons. First, entry must be limited to save on the cost of organizing and evaluating contestants. For example, when the Army failed to restrict entry into a recent procurement competition, it cost the service over $5000 just to reproduce

[1] McLaughlin, pg 248.
[2] In the U.S. Open, only the top 125 players on the money list each year are allowed "open" entry. Among other requirements, all other golf hopefuls must have a handicap of two or less and place at the top of two sequential elimination tournaments before they can even be considered for entry.

and send out bid invitations to the more than 100 firms that responded to the purchasing advertisement -- even though the equipment the Army wanted to buy was only worth a total of $11,000.[3]

Entry must also be limited to induce efficient levels of effort by the contestants who are allowed to compete for the prize. In 1983, Nalebluff and Stiglitz proved that too many workers in a labor tournament disrupts the nonconvexity constraints and results in a reduction of effort. Similarly, Taylor (1995) showed that allowing too many firms to compete in a research tournament induces mixed strategies of research resulting in an overall smaller level of effort than when a restricted number of firms engage in pure strategies of research. With too many contestants competing for the same prize, the chances of winning are so small that individual contestants are discouraged from expending effort. Thus, free and open entry is not the optimal participation policy and sponsors must devise some method for limiting entry to only the best qualified contestants.

The importance of adverse selection in tournament settings cannot be overstated because the final outcome of any tournament rests entirely on the individual abilities and incentives of the participating contestants. Failure to select

[3] The extra $5000 in taxpayers money was wasted in this case because the Army is required by law to allow all firms access to procurement competitions. The Competition in Contracting Act passed by Congress in 1984 (Public law 98-92) requires "free and open competition" in the interest of "fairness" to all sellers. It is unclear how much money DOD spends each year satisfying this requirement on inefficient competitions. See Gansler, 1989, pg. 182.

the right contestants guarantees poor final results. Even if one highly qualified contestant is allowed to participate, the tournament must also include other equally qualified contestants if the sponsor hopes to spark any meaningful competition. Thus, in sports settings where evaluation costs are low, the repeated elimination of losers in multiple-round tournaments ensures only talented contestants survive to the finals, softening the impact of adverse selection in the finals.[4]

Unfortunately, in many economic settings large evaluation costs make multiple-round elimination tournaments too expensive to use for selecting the final competitors. Fox (1974) cited a Department of Defense contract that cost the U.S. government 182,000 man-hours to evaluate the proposals of only four contractors.[5] So multiple-round elimination tournaments are impractical for many applications and sponsors must rely on other methods to select qualified contestants.

Besides Rosen's study of elimination tournaments, most of the literature on adverse selection in tournaments has revolved around enticing two types of contestants with different costs of effort to self-select into separate tournaments designed for each type.[6] This approach, while interesting, has limited real-world applicability where contestants come in a variety of types and the sponsor is only

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[4] In a study of elimination tournaments Sherwin Rosen (1986) notes that "Effort is greater in a final match involving equally talented contestants than in one which matches a stronger against a weaker player." See Sherwin Rosen, 1986, pg. 707.
[5] Fox, 1974, pg. 269.
[6] See Lazear and Rosen (1981), McLaughlin (1988), and O'Keefe, Viscussi, and Zeckhauser (1984) for discussions on inducing self-selection into separate tournaments.

conducting one tournament. This chapter addresses the more realistic problem of adverse selection when contestant types are drawn from a continuum and the sponsor wishes to select only the best qualified contestants to participate. In particular, I show that entry auctions can be ideally suited for selecting contestants in these tournaments because the best-qualified contestants value participation the most and the sponsor does not need prior information about contestant types to conduct the auction. However, I also show that the efficiency of entry auctions depends crucially on how contestants differ.

The first, and least problematic, form of contestant heterogeneity involves differences in the cost of effort. In a labor tournament these differences might be in strength, intelligence, or skills allowing some workers to complete tasks with less effort than other workers. In a research tournament this heterogeneity refers to differences reflected in a firm's cost of conducting research. The second kind of heterogeneity is differences in contestants' starting positions or technologies that place some contestants ahead of others prior to the start of the contest. For example, even if Apple Computer had the lowest costs of research, the brightest engineers, and the most efficient organization of all computer firms it would probably lose a one-year contest against sluggish IBM to build the most powerful supercomputer. While Apple would undoubtedly make large strides in its own supercomputer development, in the space of a year they would probably not be able

to produce anything that could compete with what IBM already has in production. Thus, IBM has an initial technological starting advantage over Apple which is entirely separate from their innate research costs or abilities. It is clear that both types of heterogeneity described are commonly observed in typical economic settings, but they have radically different implications for adverse selection.

The theoretical model I use for analyzing adverse selection is Taylor's (1995) model of a research tournament, though my analysis should apply equally to adverse selection problems in other settings such as labor tournaments or even athletic contests. I have chosen Taylor's model for expositional purposes because it allows explicit treatment of both kinds of contestant heterogeneity. In the following section, I briefly describe the model and show that entry auctions are efficient mechanisms for selecting contestants when firms differ only in their costs of research. By using an entry auction, the sponsor ensures the best-qualified contestants are allowed to participate in the tournament while simultaneously generating income to offset the cost of the winner's prize. In section three, to make the distinction clear between the two different forms of heterogeneity, I assume all firms have identical research costs but differ in their starting technologies. With this kind of heterogeneity, I show that there is no symmetric bidding equilibrium for the basic entry auction, even though the best-qualified contestants still value entry the most. Finally, in section four, I prove the sponsor can restore a pure-strategy

bidding equilibrium to the entry auction by making the winner's prize a function of the entry bids. Moreover, this solution is not dependent upon the distribution of innovations and can be implemented at an arbitrarily small cost to the sponsor.

1.2: RESEARCH TOURNAMENTS AND DIFFERENCES IN COST

Tournaments are useful for soliciting effort from contestants when it is impractical for the sponsor to monitor each agent's effort directly. Because the value of new innovations is often difficult to verify by outside parties and research costs are difficult to monitor, research tournaments can be used when more conventional contracting methods are fraught with moral hazard. Taylor shows an optimally designed research tournament can approach the first-best outcome if a sponsor limits participation in the contest and charges contestants an entry fee for the right to compete. Retaining his model, I assume that there are $i = 1, 2, \ldots, N$ risk-neutral firms willing to compete for a single prize which the sponsor offers to the winner of the tournament.[7]

In this section, I also assume all N firms start with the same worthless innovation, $x = 0$. To keep the analysis simple, there is only a single period in which to conduct research during the contest. Each firm which enters the contest pays research cost C and draws a single new innovation independently from the

[7] For details on Taylor's research model, see Curtis Taylor, "Digging for Golden Carrots: An Analysis of Research Tournaments," American Economic Review, June 1995.

distribution of innovations $F(x)$, which has strictly positive density over its support $[0,\bar{x}]$. To induce efficient levels of competition and save on evaluation costs, I assume the sponsor must select $M < N$ firms to compete in the contest, and commits to paying a prize, P, to the contestant which draws the largest innovation.

Originally, Taylor assumed all firms were identical with the same costs of research which was common knowledge to all parties including the tournament sponsor. By assuming all firms are identical Taylor avoided the issue of adverse selection and proved the sponsor could create an efficient tournament by randomly selecting M contestants and charging each a tournament entry fee, E, equal to:[8]

$$E = \frac{P}{M} - C$$

The unique equilibrium in this symmetric, common-knowledge game is for each invited firm to pay E and conduct research so the sponsor gets the best innovation drawn from exactly M research attempts at a cost of $P - EM = CM$. Unfortunately, if research costs are private information efficiency is not so easy to achieve.

Suppose all N firms still have identical research costs but they are private information, unknown to the tournament sponsor. Since costs are unknown to the sponsor, she has no hope of guessing the proper entry fee to charge the M randomly

[8] Taylor, Proposition 2.1.

selected contestants. If the sponsor charges too large of an entry fee, the M firms will either refuse to enter, or engage in mixed strategies of entry with fewer than M firms expected to enter in equilibrium. With too few firms, even though each will put forth substantial competitive effort, there are too few firms to conduct enough research in the time allotted for the contest. On the other hand, if the sponsor charges too small of an entry fee then clearly she could have collected more money by charging a larger entry fee, and still induced research by all M firms. Thus, when research costs are private information, fixed entry fees are inefficient even when firms have identical costs of research.

To solve this problem the sponsor could conduct an entry auction. Again, suppose all firms are identical but research costs are not known by the sponsor. If the sponsor conducts a uniform-price auction where the M highest bidders are allowed to enter the tournament by paying the (M+1)st highest bid, the sponsor can still achieve an efficient outcome. The unique equilibrium bid in the uniform-price auction is for each firm to simply bid as much as it values entering the tournament.[9] Since all firms start with identical, worthless innovations, $x = 0$, and new innovations are drawn independently from the same distribution, $F(x)$, each firm that gains entry has exactly a $\frac{1}{M}$ probability of winning the contest. Therefore, the unique

[9] See Vickrey (1961) for the original exposition of the Vickrey auction and its equilibrium. Weber (1983) discusses uniform-price auction equilibria in multiple-object auctions.

equilibrium entry bid for each contestant is: $B = \frac{P}{M} - C$ which is identical to the optimal entry fee proposed by Taylor when the sponsor knows the cost of research. Therefore, when contestants are ex ante identical, uniform-price entry auctions are efficient mechanisms for selecting tournament contestants. The uniform-price auction maximizes the sponsor's entry fee collection and ensures the proper number of firms gain entry into the tournament.

To make the problem more realistic, suppose firms are not identical because research costs differ across firms and each firm's research cost, C_i, is private information. By conducting a uniform-price entry auction the sponsor can ensure she gets the M best-qualified contestants without any information about contestant types.[10] Since each firm starts with a worthless innovation and draws new innovations from *F(x)*, given entry all firms will still have exactly a $\frac{1}{M}$ probability of winning in a one period tournament. However, since research costs differ, the lowest cost firms will make a larger net profit if they win the prize. Therefore, in a uniform-price entry auction, each firm's unique equilibrium bid is $B_i = \frac{P}{M} - C_i$ and the sponsor collects MB^* where $B^* = \frac{P}{M} - C^*$ is the bid of the (M+1)st lowest

[10] To collect the maximum entry fee, the sponsor would need to set a reserve price for the entry auction, but then M firms may not bid high enough to gain entry.

cost of all N firms. The auction is efficient as long as the sponsor offers a large enough prize to ensure $P > C^*M$. Moreover, P cannot be set too large because any amount of "excess" prize is always bid back to the sponsor through the entrant's bids. So when contestants differ only in their costs of research, auctions are always efficient if the sponsor offers a sufficiently large prize, even if the sponsor does not know the individual contestant's research costs. Since entry bids increase as research costs decrease, the entry auction ensures the most efficient firms enter the tournament. Moreover, as N increases the sponsor expects to collect larger entry fees because the expected cost of the $(M+1)$st firm falls as N rises.

Even though the sponsor has no prior information about contestant cost differences, the entry auction induces each firm to accurately report its true costs and ensures the M lowest cost firms gain entry, resulting in the highest possible entry fees for the sponsor. Therefore, when contestant heterogeneity takes the form of cost differences between contestants, by offering a sufficiently large prize auctions can be used to select the best contestants even when the sponsor has no information about contestant types. Unfortunately, this result is not robust to differences in starting positions, which I will demonstrate in the next section.

1.3: HETEROGENEOUS ENDOWMENTS IN FIXED-PRIZE TOURNAMENTS

In this and following sections I will assume that prior to the tournament each of the *N* firms is privately endowed with a single innovation, x_i, drawn independently from the distribution *F(x)*. To avoid confusion about the cause of inefficiency and differentiate starting technology differences from cost differences, I return to the assumption that research costs are common knowledge and identical across firms. Therefore, the only difference between firms is the value of the innovation, x_i , each firm holds prior to the start of the tournament. Even though firms have identical research costs in this contest, firms with the best starting innovations will still value entry the most because there is always some probability that their starting innovation will be sufficient to win the prize. Therefore, the expected value of gaining entry in the tournament is monotonically increasing in endowed innovations. To clarify our analysis, I break the discussion up into two parts, To identify the cause of the inefficiency, in part A of this section I analyze a "pure" adverse selection model where additional research is not allowed following entry in the contest. In part B, I prove that the problem of adverse selection is only worsened when firms are subsequently allowed to conduct research and improve their innovations after entering the tournament.

1.3A: Contests Without Additional Research Following Entry

In this first contest the sponsor is not attempting to induce additional research following entry in the tournament but merely wishes to procure the best product available from among all existing innovations. Perhaps due to high evaluation costs, the sponsor is forced to select $M < N$ firms to submit innovations for the sponsor's evaluation and the firm submitting the best innovation receives a predetermined prize, P.[11]

This contest is similar to the popular lobbying literature such as the model of Baye, Kovenock, and DeVries (1993). In the BKD model, a corrupt politician, caring only about the size of his campaign chest, awards a prize to one of N firms. To maximize contributions, the politician narrows the field to $M \leq N$ finalists, then conducts an all-pay auction -- awarding the prize to the firm submitting the largest nonrefundable bid, without regard to the firm's capabilities.[12] In contrast to BKD, our contest assumes politicians are genuinely interested in selecting the best firm, but narrow the field to M firms in order to reduce evaluation costs. Ultimately however, in our tournament the prize is awarded to the firm submitting the best innovation from among all *M entrants*, independently of their entry bids. Hence,

[11] We require $M \geq 2$ because if $M = 1$ the unique equilibrium is for every contestant to bid exactly P since the winner of the auction will automatically win the prize.
[12] BKD show that the only bidding equilibria for this auction are mixed-strategy equilibria. Their model differs from ours because the BKD winner is chosen solely on the basis of being the highest bidder. Our winner is selected from among finalists solely on the basis of having the best innovation without regard to bids.

ours is a "lobbying" model without the implication of corruption. Firms with better endowments actually do have a greater probability of winning the prize given entry. Therefore, our politician logically assumes the entry bids will reflect this improved chance of winning the prize and efficiently separate the firms with relatively good innovations from the firms with bad innovations.

First, suppose there is a pure-strategy bidding equilibrium in this auction and let us define $B(x_i) = b_i$ to be the unique equilibrium bid of a firm holding initial innovation x_i. If bids are increasing in starting innovations as one would expect, there will be an innovation "cutoff value" in the tournament such that any firm with an innovation, x_i, less than the cutoff value bids too little in equilibrium and is denied entry, while all firms with endowments greater than or equal to the cutoff value become entrants and are allowed to compete. To avoid confusion over notation, I define "y" to be this cutoff value. With a strictly increasing equilibrium, the probability firm i wins the prize, given that it gains entry is:

$$\Pr[win|entry] = \left(\frac{F(x_i) - F(y)}{1 - F(y)}\right)^{M-1}$$

Since endowments are independently distributed, y is an order statistic with density:[13]

[13]Note, our order-statistic notation is different from that in most math texts. The standard is to label order statistics as: $x_{1:N} \le x_{2:N} \le \ldots \le x_{N-1:N} \le x_{N:N}$. But, since we are always concerned with the M highest values of x, our ordering is: $x_{N:N} \le \ldots \le x_{2:N} \le x_{1:N}$. All of our formulas have been transformed to reflect our different notation.

$$h_{M:N}(x) = \frac{N!}{(N-M)!(M-1)!}\left[F(x)\right]^{N-M}\left[1-F(x)\right]^{M-1} f(x)$$

A firm holding x_i that bids as if it held $r > x_i$ would expect the following profit:

$$\text{(1)} \qquad \pi(r,x_i) = P\int_0^{x_i} \left(\frac{F(x_i)-F(y)}{1-F(y)}\right)^{M-1} h_{M:N}(y)dy - \int_0^{r} B(y)h_{M:N}(y)dy$$

On the other hand, a firm with x_i that bids as if it held $r < x_i$ would expect profit of:

$$\text{(1')} \qquad \pi(r,x_i) = P\int_0^{r} \left(\frac{F(x_i)-F(y)}{1-F(y)}\right)^{M-1} h_{M:N}(y)dy - \int_0^{r} B(y)h_{M:N}(y)dy$$

The first terms in (1) and (1') are the probabilities of gaining entry and winning the prize while the second terms are the expected costs of the entry bids. Using (1) , (1') and a theorem proved by Guesnerie and Laffont (1984), I show in the appendix that there is no pure-strategy equilibrium for this auction. For any differentiable pure-strategy equilibrium, the theorem by Guesnerie and Laffont requires $(\forall r)(\forall x)\ \pi(x,x) \geq \pi(r,x)\ \Rightarrow (\forall x)\ \pi_1(x,x) = 0\ \ \textit{and}\ (\forall x)\ \pi_{12}(x,x) \geq 0$. But for equations (1) and (1'): $\pi_1(x,x) = -B(x)h_{M:N}(x)\ = 0 \Rightarrow (\forall x)\ B(x) = 0$. So, the only possible pure-strategy bidding equilibrium for this auction is for all firms to bid zero. But a bid of zero by all firms is clearly not an equilibrium because then each firm has an incentive to bid infinitesimally more than zero to guarantee itself entry and a chance of winning the prize. Therefore, a strictly-increasing pure-strategy

bidding equilibrium does not exist, and the entry auction does not ensure selection of the best M contestants as finalists.[14]

Since our entry auction in this simple tournament does not have an increasing pure-strategy equilibrium, there is no reason to believe the M best firms gain entry in the contest. Unfortunately, the adverse selection problem is even more dramatic than this because the following lemmas and theorem show that there is not even a symmetric mixed-strategy bidding equilibrium for this entry auction.

Lemma 1.1: *In any symmetric equilibrium, the distribution of bids contains no mass points.*

Proof: (See Appendix 1A.) As a sketch, if there were a mass point then any firm bidding in the "mass" could significantly increase its probability of gaining entry at only an infinitesimal additional cost by raising its bid to epsilon above the mass point. Therefore, it cannot be optimal for any firm to bid at a mass point and, in particular, there cannot be a mass point at zero.

Lemma 1.2: *In any symmetric bidding equilibrium, the upper end of the support of equilibrium bids is nonincreasing in x.*

Proof: (See Appendix 1A) As a sketch, suppose there is a symmetric mixed strategy equilibrium and define $\widetilde{B}(x)$ to be the upper end of the support of the set of

[14] There is no pure-strategy equilibrium in either the uniform-price auction or the discriminatory-price auction or any combination of the two.

equilibrium bids by any firm holding innovation x. If $\widetilde{B}(x)$ is increasing, then for at least one $x^* \in [0,\bar{x}]$, $\widetilde{B}(x^*) > \widetilde{B}(x')\ \forall x' < x^*$. Suppose this is true and pick an x^* such that $\widetilde{B}(x^*) > \widetilde{B}(x')\ \forall x' < x^*$. If a firm holding this x^* bid $\widetilde{B}(x^*)$, it knows *ex ante* that it wins the prize *only if* $\widetilde{B}(x^*)$ is the largest bid because any firm submitting a larger bid by assumption also has a larger x. Since the top M firms gain entry in the tournament, the firm with x^* could have submitted a smaller bid and not reduced its probability of gaining entry when it has the largest x. Therefore, a bid of $\widetilde{B}(x^*) - \varepsilon$ is preferred to $\widetilde{B}(x^*)$ because it has a lower expected cost but still ensures entry whenever the firm has the best innovation. Thus, $\widetilde{B}(x^*)$ cannot be an equilibrium bid which implies $\forall x^* \in [0, \bar{x}]$, $\exists\, x' < x^*$ such that $\widetilde{B}(x') \geq \widetilde{B}(x^*)$ and therefore the maximum bid is nonincreasing and the maximum equilibrium bid by all firms can be no larger than the maximum equilibrium bid of the firm with $x = 0$.

Lemma 1.3: *In any symmetric bidding equilibrium, all firms must bid 0.*

Proof: (See Appendix 1A) As a sketch, by assumption the sponsor does not accept negative bids.[15] By definition, there are $M \geq 2$ entrants. Since $F(x)$ is continuous, there is zero probability of a tie at $x = 0$. Therefore, $x = 0$ has zero probability of

[15] We assume the sponsor does not accept negative bids because any firm with a positive x can always submit a bid of zero and win the prize with their initial endowment without doing any research. With negative bids, the sponsor is giving away money without ensuring additional research is conducted so negative bids are weakly dominated by bids of zero for the sponsor.

winning the prize even if it gains entry, so it bids zero to avoid paying an entry fee. From lemma 1.2 the maximum bid of $x = 0$ is at least as large as the maximum bid of all other firms, therefore all firms must bid 0 in any equilibrium.

Lemma 1.2 provides the necessary intuition for why there is no symmetric equilibrium in entry auctions with different starting innovations: nobody wants to be the marginal entrant! If a firm is the "marginal entrant," it must pay the entry fee but has no hope of winning the prize. Therefore, for marginal firms it is better to bid zero and avoid paying any entry fee at all than to pay an entry fee and not win the prize. But then even firms with very good endowments will know marginal firms prefer to bid zero, so this induces the best firms to lower their bids as well and there is no symmetric equilibrium.

THEOREM 1.1: *A symmetric equilibrium does not exist for the entry auction in a fixed-prize procurement contest if firms differ in their initial endowments and cannot improve their innovations following entry.*

PROOF: Suppose there is a symmetric bidding equilibrium. Lemma 1.1 allows no mass points at a bid of zero. Lemma 1.3 requires all firms to bid zero. This is a contradiction. Q.E.D.

In the BKD contest and other lobbying models, there are generally a continuum of mixed-strategy equilibria.[16] But in lobbying models the prize winner is simply the firm that submits the largest bid. In contrast, a larger entry bid in our tournament only influences your probability of entry but, given entry, the bid is irrelevant to a firm's probability of winning. A firm cannot win the prize simply by bidding more -- regardless of how high it raises its bid. This difference between the two models is sufficient to destroy all symmetric bidding equilibria in our research contest.

To illustrate, consider a special example where x is allowed to take on only integer values: $x \in \{0,1,2,3, \ldots\}$. Now consider the bid of a firm holding some value x_i assumed to be the smallest value of x for which a firm prefers to submit a strictly positive bid in order to ensure itself entry when all other N firms are smaller. Since a continuum of bids are allowed and a higher bid does not improve one's chance of winning given entry, the optimal bid is equal to "the smallest possible positive bid". Of course there is not a number corresponding to "the smallest positive bid", so there is no equilibrium bid for this firm. No matter what bid x_i chooses, there is always a smaller bid which is still larger than zero, and therefore strictly better than the bid it chose. Hence, there is no equilibrium bid for x_i , and there is no equilibrium for the auction. Notably, if we restrict bids to only

[16] Baye et al., 1993, pg. 289.

"discrete" units, then the auction does have a unique pure-strategy bidding equilibrium. But, even then the equilibrium is not efficient because it requires significant probabilities of tie bids among firms-- so the sponsor still does not ensure that she gets the best qualified firms with the best innovations to enter the tournament.[17]

Even though the "honest" politician in our model genuinely wanted to serve the public and pick the firm with the best innovation, any entry auction the politician selects is inefficient and does not ensure the best firm wins the prize. With no symmetric equilibrium, it is unclear which firms will be selected for entry and our sponsor may do no better than the "corrupt" politician in the BKD lobbying model. Unfortunately, this problem is not resolved by allowing firms to conduct research after entering the tournament.

1.3B: Contests With Additional Research Following Entry

Suppose finalists can conduct research after being selected for entry. Firms with the best initial endowments still have the greatest probability of winning the prize, but now lesser endowed firms may also win the prize by drawing a winning innovation after gaining entry. Following Taylor, assume there are $T > 0$ periods following entry for each firm to conduct research. In each period, any contestant

[17] For a discussion of the discrete auction, see appendix 1C.

may pay research cost C for a single independent draw from the distribution of new innovations, *F(x)*. Firms are only allowed to conduct research once per period and research costs, *C*, are assumed to be the same for all firms. To focus attention on research with heterogeneous endowments, I will assume *C* is common knowledge to all players including the sponsor. After every draw, each firm retains its best innovation, which is private information, and at the end of *T* periods, each finalist submits its best innovation to the sponsor for evaluation. The prize is awarded to the firm submitting the best innovation, there is no time discounting and all firms are assumed to have sufficient capital to conduct research in every period if they desire.

Given entry, Taylor proves there is a unique, subgame perfect "*z-stop*" research strategy for each contestant. Let $\Pi_i(x)$ be the CDF of the best innovation ultimately discovered by any of firm *i*'s rivals. If firm *i* stops doing research after obtaining a draw of *x*, it's expected profit (gross of sunk costs) is $P\Pi_i(x)$. On the other hand, the expected profit from performing one more round of research is:[18]

$$P\Pi_i(x)F(x) + P\int_x^u \Pi_i(y)dF(y) - C$$

Therefore, firm *i*'s net expected profit of conducting another round of research is:[19]

$$D_i(x) = P\int_x^u [\Pi_i(y) - \Pi_i(x)]dF(y) - C$$

[18]Taylor, pg 11.
[19]Taylor, pg 11.

Taylor shows that firm i's optimal strategy is to conduct research in each period until drawing an innovation worth at least z_i, implicitly defined by $D_i(z_i) = 0$. Taylor refers to this optimal stopping strategy by entrants as a *z-stop* strategy. Since all finalists draw independently from the same distribution and have the same costs of research, z is unique and identical for all finalists.[20] If I again define "y" to be the cutoff value of innovations by firms gaining entry in the tournament, the probability a contestant's final best innovation is no better than some arbitrary innovation x is:

$$\Phi(x,y,z(y),T) = \begin{cases} 0 & \text{for } x<y \\ \left(\dfrac{F(x)-F(y)}{1-F(y)}\right)F^T(x) & \text{for } y<x<z \\ \left(\dfrac{F(x)-F(z)}{1-F(y)}\right)+\left(\dfrac{F(z)-F(y)}{1-F(y)}\right)\left[F^T(z)+(1-F^T(z))\left(\dfrac{F(x)-F(z)}{1-F(z)}\right)\right] & \text{for } x>z \end{cases}$$

To explain this distribution, any firm with an initial endowment $x < y$ does not gain entry in the tournament, so the top term is zero. For any contestant whose best final innovation, x, is such that $y < x < z$, the contestant will conduct research in all T periods because x is less than the *z-stop* which explains the middle term. Finally, for $x > z$, the first term on the bottom gives the probability of having an initial endowment better than z, and the second term is the probability of starting with an initial innovation worse than z but subsequently drawing an innovation better than z

[20] Taylor, proposition 2.2

and stopping research. In this tournament, z is a function of y because a larger y implies a higher expectation of the best initial endowment.

Lemma 1.4: *The unique value where $z = y$ is given by the equation:*

$$F(z) = F(y) = 1 - \frac{MC}{P}$$

Proof: (See Appendix 1A)

Given C and P, the intersection of the stopping value, z, and the cutoff value, y, decreases as the number of finalists increase. This equation provides a simple gauge of the effect that raising the number of contestants has on the equilibrium level of effort of all contestants. To illustrate, in Figure 1.1, I graph z and y for the uniform [0,1] distribution, a prize/cost ratio of 10, T = 1 and varying values of M:

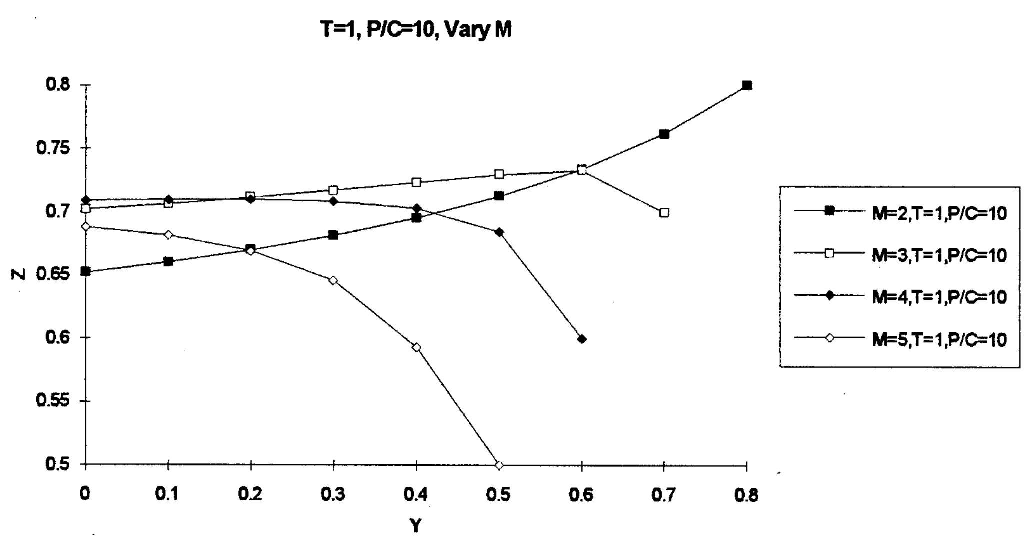

Figure 1.1

Only when $M = 2$ is the equilibrium value of z strictly increasing in y (See Appendix 1A for proof.) As can be seen, when $M > 2$ an increase in y can lead to a decrease in z. In other words, with more than two firms, a larger "cutoff value" (corresponding to better initial endowments) may lower the equilibrium level of z, the stopping value for future research.

From an individual firm's perspective, a higher value of z has two opposing effects: a larger z-stop increases the probability the firm will eventually draw an innovation that exceeds the best initial innovation of the other M-1 finalists, but raising z also increases the likelihood that other finalists will draw better innovations. When $M > 2$ the second effect dominates the first because each firm faces more than one competitor. The only means of ensuring that the equilibrium level of effort (measured by z) rises as y increases is to restrict competition to just two finalists.

By allowing firms to conduct research after entry, we do not restore equilibrium or efficiency to the entry auction, instead we only exacerbate the problem. Let us define x_1 to be a firm's initial endowment and define x_T to be the best innovation other than x_1 that the firm draws during its research efforts after entering the tournament. Therefore, at the conclusion of the tournament the firm submits either x_1 or x_T, whichever is highest, to be evaluated by the sponsor. By doing this, we can partition each firm's expected profits according to its probability

of winning with x_1 and its probability of winning with x_T. Given entry, a better initial endowment increases the probability a firm wins with x_1 but decreases the probability the firm will win with x_T. In particular, if a firm's initial endowment is larger than the equilibrium *z-stop* value it will not conduct any research following entry and the firm's probability of winning with x_T is zero.

From Lemma 1.4, whenever $P < \infty$ and $C > 0$, the maximum possible value of z is strictly less than $\bar{x}$. Therefore, any firm holding an initial endowment x_1 that falls in the interval $\bar{x} > x_1 > z_{max}$ will *never* conduct research following entry in the tournament. Since these best-qualified firms never conduct research after entering the tournament, each of them sees the tournament as essentially equivalent (from their perspective) to the previous contest without research. As I have already shown, the only possible pure-strategy bidding "solution" for firms in that contest was to bid zero. That result also carries through in this contest for these firms that will not conduct research after entry. Therefore, since the firms with the largest initial endowments bid zero, there clearly cannot be a strictly-increasing pure-strategy bidding equilibrium. This, implies that entry auctions do not ensure the best-qualified firms are selected as finalists in tournaments where research is allowed following entry.

Proposition 1.1: *Entry auctions are inefficient mechanisms for selecting contestants in multiple-period, fixed-prize tournaments when contestants have different initial endowments.* (See appendix for proof.)

To demonstrate Proposition 1.1, consider the following tournament where M = 2 and T = 1. Once again I appeal to the theorem by Guesnerie and Laffont to solve for the "optimal" bids and refer the reader to the appendix for details. Whenever there are two finalists and exactly one additional period of research allowed following entry, the *ex ante* expected profit of any firm holding endowment x_1 is given by:

$$P\int_0^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)\left\{\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)+\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)F(x_1)\right\}h_{M:N}(y)dy$$

$$+\ P\int_{z^{-1}(x_1)}^{x_1}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)[F(x_1)]^2\,h_{M:N}(y)dy$$

$$+\ P\int_{z^{-1}(x_1)}^{x_1}\left[\int_{x_1}^{z(y)}\left(\frac{F(x_2)-F(y)}{1-F(y)}\right)F(x_2)f(x)dx_2\right]h_{M:N}(y)dy$$

$$+P\int_{z^{-1}(x_1)}^{x_1}\left[\int_{z(y)}^{\bar{x}}\left(\frac{F(x_2)-F(y)}{1-F(y)}\right)\right.$$

$$\left.\times\left\{\left(\frac{F(x_2)-F(z(y))}{F(x_2)-F(y)}\right)+\left(\frac{F(z(y))-F(y)}{F(x_2)-F(y)}\right)F(x_2)\right\}f(x)dx_2\right]h_{M:N}(y)dy$$

$$-\int_0^{x_1}B(y)h_{M:N}(y)dy\ -\int_{z^{-1}(x_1)}^{x_1}C\,h_{M:N}(y)dy$$

The first two terms are the firm's expected profits from its initial endowment. The 3rd and 4th terms are the expected profits from a new draw, x_2, given that $x_1 < z$ and $x_2 > x_1$. The last two terms are the expected costs of the bid and research respectively. Applying the theorem by Guesnerie and Laffont, the only possible pure-strategy bidding function is:

$$B(x_1) = P \int_{x_1}^{z(x_1)} \left(\frac{F(x_2) - F(x_1)}{1 - F(x_1)} \right) F(x_2) f(x) dx_2 + P \int_{z(x_1)}^{\bar{x}} \left(\frac{F(x_2) - F(z(x_1))}{1 - F(x_1)} \right) f(x) dx_2$$

$$+ P \int_{z(x_1)}^{\bar{x}} \left(\frac{F(z(x_1)) - F(x_1)}{1 - F(x_1)} \right) F(x_2) f(x) dx_2 \ \text{- C}$$

This cannot be an equilibrium, however, because *B(x)* is *decreasing* in initial endowments. (See Appendix 1B)

Why such an odd result with inverted bids? Suppose bids were strictly increasing in endowments. If a well endowed firm gains entry, it has learned relatively nothing about it's expected competition since it always expects to gain entry. In contrast, a poorly endowed firm does not expect to gain entry in the tournament unless the critical value y is very low -- implying the "average" quality of its competition is poor. Since research is allowed following entry, knowing that one only gains entry when other finalists are of poor average quality creates the expectation that a firm has a good chance of winning with an innovation drawn after entry. This applies pressure for poorly endowed firms to raise their bids. As a

result, if we *assume* bids are increasing in endowments, and then solve for the entry bids as a function of x_1, the bidding function is inverted. In Figure 1.2, I plot the suggested bidding equilibrium to illustrate the bid inversion for a tournament when *F(x)* is uniform [0,1], T = 5, M = 5, P = 20, and C = 1. (See Appendix 1B)

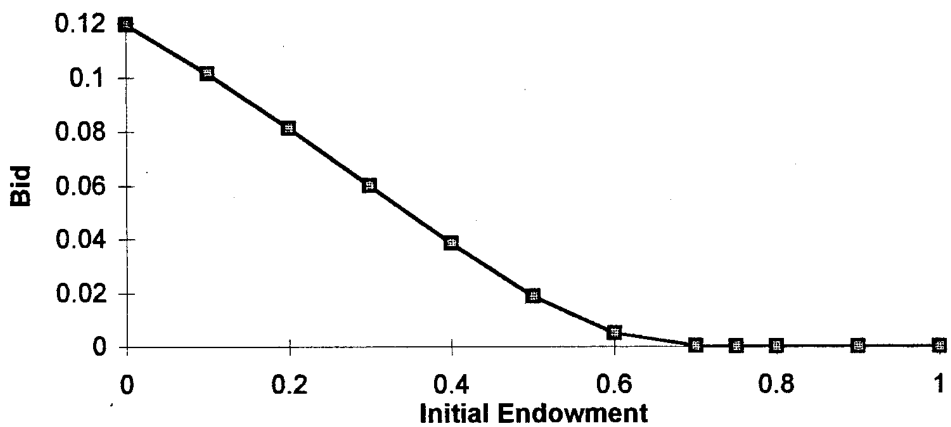

Figure 1.2

Clearly, the bidding scheme in the figure above is not an equilibrium, but it demonstrates the "bid inversion" described in the previous paragraph, and suggests why restoring efficiency to a multiple-period tournament is more difficult when firms are allowed to conduct research following entry. Therefore, when contestants have different initial endowments of technology the use of an entry auction is inefficient and does not ensure the sponsor obtains the best-qualified firms for the tournament. In the following section, I offer a solution for restoring a strictly increasing equilibrium to entry auctions by tying the value of the tournament prize to the size of the entry bids.

1.4: RESTORING EQUILIBRIUM TO ENTRY AUCTIONS

To restore equilibrium to the entry auction, we must provide an incentive for better-endowed firms to submit increasingly larger bids. By awarding larger prizes to firms which submit higher bids, we make it worthwhile for firms with better innovations to risk bidding more, because they get a larger payoff if they win. To illustrate, consider again the contest without research following entry. When the prizes are a function of the bids in a uniform-price auction the expected profits are:

$$(2)\ \pi = P\big(B(x_i)\big) \int_0^{B^{-1}(b)} \left(\frac{F(x_i) - F(y)}{1 - F(y)}\right)^{M-1} h_{M:N}(y)dy - \int_0^{B^{-1}(b)} B(y)h_{M:N}(y)dy$$

This function is maximized by any bidding function satisfying the following equation:

$$P'(b)B'(x_i)F(x_i)^N = B(x_i)h_{M:N}(x_i)$$

Substituting for $h_{M:N}$ and letting $K = \dfrac{N!}{(N-M)!(M-1)!}$ this equation reduces to:[21]

$$P'(x) = \frac{B(x)K[1-F(x)]^{M-1} f(x)}{F(x_i)^M}$$

A solution to this equation that works for *all* distributions is:

$$P(b) = \frac{K}{M} - \frac{K}{M}\left[1 - b^{1/M}\right]^M \quad \text{and} \quad B(x) = F(x)^M$$

[21]Note that P'(x) = P'(b)B'(x)

This solution satisfies the criteria of bids being strictly increasing in x and $B(0) = 0$. The most important thing to notice about this solution, however, is that it works for any distribution of innovations, $F(x)$. By making the prize an increasing function of the bidding, the tournament sponsor can always restore efficiency to the entry auction. Furthermore, the sponsor can choose any positive real number, L, and arbitrarily scale both the prizes and the bids to form another equilibrium:

$$P(b) = \frac{K}{LM} - \frac{K}{LM}\left[1 - (Lb)^{\frac{1}{M}}\right]^{M} \quad \text{and} \quad B(x) = \frac{F(x)^{M}}{L}$$

As we raise L the prizes become smaller, the bids become smaller, and the average slope of the equilibrium bidding line is shallower. In Figures 1.3 and 1.4, I graph the prizes and equilibrium bids for a tournament where endowments are distributed uniformly on the [0,1] interval, with $M = 2$, $N = 10$. In order to achieve a contrast, I pick values of L = 5 and L = 10.

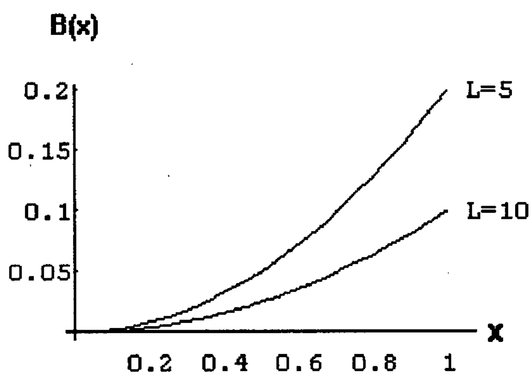

Figure 1.3

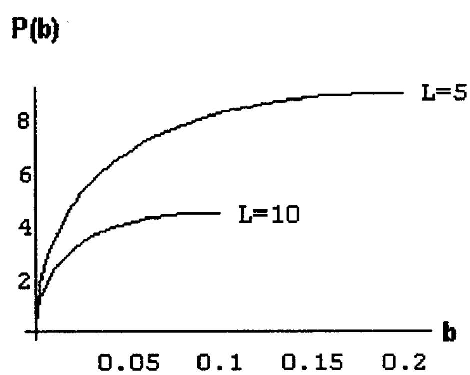

Figure 1.4

By choosing a sufficiently large number for L, the sponsor can restore efficiency in the $T = 0$ procurement contest at an arbitrarily small cost. One can use the same technique to restore equilibrium when research is allowed after entry and T > 0. The following expression implicitly defines the bidding for $M = 2$ and $T = 1$.

$$
\begin{aligned}
B(x_1)h(x_1) = P'(x_1) &\int_0^{z^{-1}(x_1)} \left(\frac{F(x_1)-F(y)}{1-F(y)}\right) \\
&\times \left\{\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)+\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)F(x_1)\right\} h_{M:N}(y)dy \\
+P'(x_1) &\int_{z^{-1}(x_1)}^{x_1} \left(\frac{F(x_1)-F(y)}{1-F(y)}\right)\left[F(x_1)\right]^2 h_{M:N}(y)dy \\
+P'(x_1) &\int_{z^{-1}(x_1)}^{x_1} \left[\int_{x_1}^{z(y)} \left(\frac{F(x_2)-F(y)}{1-F(y)}\right)F(x_2)f(x)dx_2\right] h_{M:N}(y)dy \\
+P(x_1) &\left[\int_{x_1}^{z(x_1)} \left(\frac{F(x_2)-F(x_1)}{1-F(x_1)}\right)F(x_2)f(x)dx_2\right] h_{M:N}(x_1) \\
+P'(x_1) &\int_{z^{-1}(x_1)}^{x_1} \left[\int_{z(y)}^{\bar{x}} \left(\frac{F(x_2)-F(y)}{1-F(y)}\right)\right. \\
&\left.\times \left\{\left(\frac{F(x_2)-F(z(y))}{F(x_2)-F(y)}\right)+\left(\frac{F(z(y))-F(y)}{F(x_2)-F(y)}\right)F(x_2)\right\} f(x)dx_2\right] h_{M:N}(y)dy \\
+P(x_1) &\left[\int_{z(x_1)}^{\bar{x}} \left(\frac{F(x_2)-F(x_1)}{1-F(x_1)}\right)\right. \\
&\left.\times \left\{\left(\frac{F(x_2)-F(z(x_1))}{F(x_2)-F(x_1)}\right)+\left(\frac{F(z(x_1))-F(x_1)}{F(x_2)-F(x_1)}\right)F(x_2)\right\} f(x)dx_2\right] h_{M:N}(x_1) - Ch_{M:N}(x_1)
\end{aligned}
$$

All that is required to restore a strictly increasing equilibrium to the equation above is for $P'(x)$ to be sufficiently large to make $B(x)$ an increasing function of x. As discussed earlier, it is more costly to restore equilibrium when $T > 0$ because $P'(x)$ must be large enough to overcome the effects of the fourth and sixth terms which include $P(x)$ and get smaller as x increases, reflecting the "bid inversion" examined in section 3.

1.5: CONCLUSION

Adverse selection is a formidable problem in tournaments if the sponsor has limited information about contestant types. However, for some types of heterogeneity, simple entry auctions ensure the sponsor gets the best contestants and collects the maximum amount of entry fees possible given limited information. This result is not robust to all kinds of contestant differences. When heterogeneity takes the form of differing endowments, there is no efficient bidding equilibrium for entry auctions in tournaments with fixed prizes. This suggests that tournament sponsors should consider altering their prize structure if they wish to use auctions to select contestants for tournament participation. By making the contest prize an increasing function of entry bids, sponsors can restore efficiency to the contestant selection process and ensure only the best qualified contestants participate in the competition. Whenever $T = 0$, a strictly increasing bidding equilibrium can be restored at virtually

no cost to the sponsor. For tournaments with additional research opportunities following entry, the sponsor must weigh the benefits of ensuring he gets the best qualified contestants against the cost of raising the prize sufficiently to induce a strict ordering of bids by ability. Ultimately, however, this study suggests that auctions can be usefully incorporated into contests to reduce or eliminate the problem of adverse selection which has plagued the tournament literature since its beginnings.

Chapter 2

Tournaments with Prizes Determined by Auction

This chapter compares research tournaments offering fixed prizes with a proposed auction-style tournament in which the prize is determined by contestant bids at the end of the contest. I prove the auction-style tournament has a unique symmetric equilibrium that induces the same level of effort by contestants at a lower expected cost than the equivalent fixed-prize tournament. Additionally, the sponsor needs less information to implement the auction-style tournament and never risks paying a larger prize than the innovation is worth, so the auction-style tournament dominates fixed-prize tournaments for conducting research and development competitions.

2.1: INTRODUCTION

Contracting directly for research and development is extremely difficult because of the uncertainties involved in discovering and applying new technologies and the problem of accurately observing contractor efforts and expenses. Studying these problems in defense contracting, William Rogerson[1] argued that the government must give agents an incentive to exert effort by promising to reward successful innovations with "prizes." A regulatory structure that directly rewards agents with larger prizes for better innovations would be optimal, but Rogerson suggests it may not be feasible because the value of innovations cannot be objectively verified by an impartial court. Therefore, any prize would always be subject to dispute by at least one party to the contract.

To solve the problem of contracting for research, Curtis Taylor suggested creating a research tournament.[2] In Taylor's tournament, firms conduct research to compete for a predetermined prize offered by the tournament sponsor. At the beginning of the tournament, the sponsor announces a prize to be awarded to the firm producing the best innovation at some specified date in the future corresponding to the end of the research tournament. Along with the prize announcement, the sponsor reveals her innovation preferences so firms know which kinds of innovations are preferred by the sponsor. After entering the tournament,

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[1] Rogerson, *JPE, 1989.*
[2] Taylor, 1995.

firms may conduct research in each time period, as desired, until the end of the tournament. On that final date, each firm submits its best innovation, the sponsor picks her favorite, and the firm producing the winning innovation is given the prize.

Verifiability is not an issue in Taylor's tournament, because the courts need only observe whether or not the prize is actually paid at the designated time. Since prize payment is easily verified by a court, firms are not subject to hold up by the sponsor. However, one problem emphasized by Rogerson that is not resolved by Taylor's research tournament is the ability to award larger prizes to firms for better innovations.

Under Taylor's fixed-prize tournament, the winner is given the same prize regardless of the quality of its winning innovation. Additionally, the fixed-prize tournament places a substantial information burden on the sponsor to figure out the proper size of the prize to induce efficient levels of effort. It is unlikely a sponsor will ever know enough about all emerging technologies and individual firm capabilities to accurately compute the efficient prize. Optimally, we would like to reduce the sponsor's information burden while simultaneously incorporating a prize structure that rewards firms with larger prizes for better innovations.

Fortunately, this is feasible if the sponsor conducts an "auction" at the end of the tournament. In this new auction-style tournament, firms submit not only their winning innovations at the end of the contest, but they also simultaneously submit

sealed bids or prices which they would charge the sponsor for purchasing their innovation. Each firm sets its own price based on the value of the innovation it submits. In this way, firms with lesser quality innovations can make their products more competitive by submitting lower bids, and firms with high-quality innovations can submit bids which charge a premium for their better quality. In equilibrium, firms which develop the best innovations expect the largest profits. This increases the incentives for firms to continue researching -- raising the equilibrium level of effort by firms. Therefore, this new auction-style tournament requires *less* information be known by the sponsor, yet it induces greater equilibrium levels of effort by firms for the same expected cost. By wisely incorporating an impartial third party, I also show that an equilibrium exists where the sponsor can achieve all of the cost-saving benefits of the auction-style tournament without losing the ability to independently control the equilibrium level of effort put forth by tournament contestants.

2.2: EQUILIBRIUM BIDS IN THE AUCTION-STYLE TOURNAMENT

To demonstrate the dominance of the auction-style tournament, I use a model that is identical to Taylor's research tournament except for the form of the prize.[3] Following Taylor's original notation, there are $M \geq 2$ risk neutral firms

[3] The reader is directed to Taylor, 1995, for details regarding the research tournament model.

(contestants) which the sponsor invites to participate in the contest. There are $T \geq 1$ periods in which each firm has the opportunity to conduct research. Each firm can conduct research once per period, and if a firm chooses to conduct research, it pays research cost C and gets a single *independent* draw, x, from the innovation distribution $F(x)$. The distribution of innovations is defined on $[0,U]$ with $F(0)=0$ and $U \leq \infty$. Innovation draws are independent across time and firms. Each firm retains its best draw and discards all other draws as being worthless. The value of innovations is private information as is the very act of conducting research.

Taylor's tournament has the M entrants competing for a single predetermined fixed prize, P. If we assume x is the best innovation submitted at the end of the Taylor tournament, the sponsor's net surplus is: $x - P$. In the auction-style tournament, at the end of T periods each firm i submits its best draw, x_i, along with a bid, $B(x_i)$, and the sponsor selects the firm which offers the largest net surplus: $x - B(x)$.

Our first priority is to solve for the optimal bidding function by firms in the final auction. Following Taylor, we assume the actual cost of production of all innovations is identical and equal to zero. This is without loss of generality because if there were different costs of production, we could scale the x values and the bids appropriately to reflect those differences. Excluding the sunk costs of research, the

winning firm's prize is just equal to its final bid. Therefore, each firm "i" holding some final innovation x_i chooses its bid $B(x_i)$ to maximize:

$$\max_{B(x)} \; B(x_i)\Pr[x_i - B(x_i) > x_j - B(x_j)] \quad \forall\, i \neq j\,.$$

We would like a pure-strategy bidding function in which the surplus, $x - B(x)$, and the bids are strictly increasing in x. This would imply that the firm with the largest value of x (i.e., the best innovation) is the tournament winner, and expected profits are increasing in innovations. Additionally, we would like the bidding function to be unique. Fortunately, these are the properties of the standard Independent Private Values (IPV) auction solution, and this auction is a simple transformation of the standard IPV auction.

To see this, note that in a typical IPV auction, firms draw x from $F(\text{x})$ and submit bids $b(x)$ to maximize $\{x_i - b(x_i)\}\Pr[b(x_i) > b(x_j)] \quad \forall\, i \neq j$. The unique solution to this IPV auction is for each firm to submit a bid of[4]:

$$b(x_i) = x_i - \frac{\int_0^{x_i} [F(\xi_i)]^{M\text{-}1} d\xi}{[F(x_i)]^{M\text{-}1}}$$

Making the transformation $B(x) = x - b(x)$, firms in an IPV auction are maximizing: $B(x_i)\Pr[x_i - B(x_i) > x_j - B(x_j)] \quad \forall\, i \neq j.$ (which is our auction) and the optimal bid for the auction-style tournament becomes:

[4] See McAfee, Preston and John McMillan. "Auctions and Bidding" *Journal of Economic Literature*, June 1987, vol. XXV. pp 699-738.

$$B(x_i) = \frac{\int_0^{x_i} [\Phi(\xi_i)]^{M\text{-}1} d\xi}{[\Phi(x_i)]^{M\text{-}1}}$$

($\Phi(\xi)$ and $\Phi(x)$ represent the distribution of best final innovations drawn by firms)

This optimal bidding function is unique and strictly increasing in both x and $[x\text{-}B(x)]$. Therefore, if all firms submit equilibrium bids at the end of the tournament, expected profits will be increasing in innovations, and the firm with the best draw will win the tournament. Thus, the auction-style tournament preserves the nice monotonic features of Taylor's original fixed-prize tournament.

2.3: THIRD PARTY PARTICIPATION

One problem with auction-style tournaments is the apparent lack of flexibility the sponsor has in adjusting the winner's prize. In particular, the only choice variable for the sponsor is M. This can be a drawback of auction-style tournaments because the sponsor may prefer to induce more or less effort from contestants than the equilibrium level of effort given M.

One might try to fix this problem by conducting a hybrid tournament where the sponsor promises to award a lump-sum prize in addition to the bid of the winning contestant. Unfortunately, this plan will not work because then the optimal bid for each contestant just becomes the previous equilibrium bid adjusted

downward by the exact amount of the lump-sum prize. Similarly, if the sponsor announces *ex ante* that the winning contestant will be paid a multiple of some arbitrary constant "*K*" times his winning bid, each contestant knows the sponsor will have an *ex post* incentive to maximize $x - KB(x)$. Therefore, each contestant solves:

$$\max_{B(x)} \; KB(x_i)\Pr[x_i - KB(x_i) > x_j - KB(x_j)] \quad \forall\, i \neq j\,.$$

By making the transformation $KB(x) = \hat{B}(x)$, one can see this problem is equivalent to:

$$\max_{\hat{B}(x)} \; \hat{B}(x_i)\Pr[x_i - \hat{B}(x_i) > x_j - \hat{B}(x_j)] \quad \forall\, i \neq j\,.$$

implying that each contestant adjusts their bid by a factor of *K* so the prize structure and equilibrium level of effort remains unaltered by the sponsor.

However, if the sponsor incorporates an impartial third party into her tournament, she may be able to alter the prize structure and change the equilibrium effort level. Moreover, the third party need not have any technical knowledge, so the third party could be a court. The third party solution for altering the prize structure works as follows:

STAGE 1: The sponsor selects contestants and announces "K" along with a third party (e.g., the court) to retain all bids until evaluations have been made.

STAGE 2: Contestants conduct research and submit their best innovations to the sponsor, while submitting their *sealed* bids to the third party.

STAGE 3: Without seeing the sealed bids (now in possession of the courts) the sponsor judges each of the innovations submitted by the contestants and informs the courts of the sponsor's value of each innovation (i.e., "x").

STAGE 4: The court, having received the x valuation from the sponsor (a dollar figure), compares $x - B(x)$ for each firm and declares the winner to be the contestant offering the largest net surplus. The winning firm is paid its bid and the sponsor keeps the winning innovation.

Notice the subtle difference between this tournament with a third party and the previous analysis without the third party. By including a third party, the sponsor is able to commit herself *ex ante* to maximizing $x - B(x)$ *ex post.* This is possible because the sponsor cannot actually see the contestants' bids at the time she evaluates final innovations, even though she knows what they are in equilibrium. As long as "K" is chosen to maintain the monotonicity of $x - KB(x)$, the sponsor has no incentive to lie about the value of each contestant's innovation. The sponsor maximizes $x - B(x)$, and contestants maximize:

$$\max_{B(x)} \; KB(x_i)\Pr[x_i - B(x_i) > x_j - B(x_j)] \quad \forall\, i \neq j\,.$$

Since K is a constant, the equilibrium bids of each contestant are unaltered and the sponsor can manipulate the size of the prize and the equilibrium level of contestant effort.

We should remember that nobody is fooled in this contest. Even though a third party is holding the final bids, the sponsor still knows what those bids are in equilibrium by evaluating each contestant's x. However, since x - $KB(x)$ is increasing in x and the sponsor cannot actually see deviations from the equilibrium bids, firms cannot subvert the sponsor's incentives and gain by changing their bids and the sponsor cannot gain by lying to the third party about the value of innovations. As long as K is chosen so that it does not break the monotonicity of the term x - $KB(x)$, this policy allows the sponsor to restructure the prizes without changing the number of contestants.

What values of K are acceptable? We know K must be chosen to ensure that x - $KB(x)$ is monotonically increasing in x. Therefore, we require:

$$\frac{\partial\left[x - KB(x)\right]}{\partial x} \geq 0$$

Substituting for $B(x)$ and taking the derivative with respect to x, our *ex post* incentive compatibility requirement for the sponsor requires:

$$K \leq \frac{1}{1-(M-1)\dfrac{B(x)\phi(x)}{\Phi(x)}}$$

Any $K \leq 1$ is always acceptable, so the sponsor can always reduce the equilibrium level of effort. (Note when K = 1, this is the same as an unaltered auction.) But the upper bound on K depends on the distribution of innovations and the number of

contestants. For example, if the final distribution of innovations is uniform[0,1], then the equilibrium bid of each contestant is just x/M and the acceptable range of K is just: $K \leq M$. This makes good common sense because if $K > M$ the sponsor would suffer a loss in the tournament. Thus, I have shown that there is an equilibrium in which the sponsor reports the true value of each innovation, x, to the third party, thereby permitting the sponsor to alter the prize structure and manipulate the equilibrium level of effort by contestants.

Unfortunately, this third party equilibrium is weakly dominated by the sponsor reporting x/K instead of x. If firms bid $B(x)$ as discussed, then whether or not the sponsor reports the true values of x or the altered value of x/K does not matter because K was chosen to maintain the monotonicity of both $x- KB(x)$. Thus, if all firms submit equilibrium bids, the sponsor is indifferent between reporting x and x/K. However, if some firm submits a bid off the equilibrium, then by reporting x/K to the third party the sponsor makes the third party maximize $x/K - b(x)$, which is the same as maximizing $\frac{1}{K}\left[x - Kb(x)\right]$ which selects the same firm as if the sponsor were maximizing $\left[x - Kb(x)\right]$. Therefore, submitting a bid of x/K weakly dominates submitting a bid of x from the sponsor's point of view -- which potentially destroys the value of the third party option for altering the equilibrium level of effort.

In many tournament scenarios, such as government research and development projects, the same firms participate for different contracts over extended periods of time. In these repeated tournament settings the sponsor may be willing to play a weakly dominated strategy in any single tournament in order to establish a reputation that allows her to manipulate effort levels in future tournaments.

2.4: TOURNAMENT COST COMPARISONS

Taylor proved the optimal strategy of contestants in a fixed-prize research tournament is a "*z-stop*" strategy. This type of strategy is a standard result of search theory in which each firm continues to pay research cost C and make research draws until it obtains an innovation worth at least "z". If a firm stopped conducting research prior to drawing an innovation worth z or better, it could increase its expected profit by drawing again since the net expected value of making another draw exceeds the expected value of its current best draw.

Following Taylor's model, I define $\Pi_i(x)$ to be the CDF of the best innovation ultimately discovered by any of firm *i's* rivals. Taylor shows that the optimal value of z for a contestant in a fixed-prize research tournament is implicitly defined by the equation:[5]

[5] Taylor, equation (2).

$$P\int_{x}^{U}[\Pi_i(y)-\Pi_i(x)]f(y)dy-C=0$$

Since *B(x)* is unique and monotonically increasing in *x*, the *z*-stop for the auction-style tournament is implicitly defined by the corresponding equation:

$$\int_{x}^{U}[KB(y)\Pi_i(y)-KB(x_i)\Pi_i(x)]f(y)dy-C=0$$

Taylor shows the date-zero CDF for the value of a firm's best innovation drawn is:[6]

$$\Phi(x;z,T)=\begin{cases}[F(x)]^T & x\le z\\ \\ [F(z)]^T+\left[1-F^T(z)\right]\dfrac{F(x)-F(z)}{1-F(z)} & x>z\end{cases}$$

This CDF has an associated density of:

$$\phi^L(x)=T[F(x)]^{T-1}f(x) \qquad x\le z$$
$$\phi^U(x)=\frac{1-F^T(z)}{1-F(z)}f(x) \qquad x>z$$

Since each firm faces M-1 other competitors and identical firms follow the same *z-stop* strategy, the distribution of the best draw of all of firm *i's* rivals is just: $\Pi_i(x)=\Phi^{M-1}(x{:}z,T)$. Substituting this combined CDF into the previous equations defining the *z-stop* we have the two corresponding equations defining the *z-stops* for the fixed-prize tournament (1P) and the auction-style tournament (1A):[7]

[6] Taylor, equation (5)
[7] Taylor, equation (8)

(1P) $$P\int_{z}^{U}\left[\Phi^{M-1}(y;z,T)-\Phi^{M-1}(z;z,T)\right]f(y)dy-C=0$$

(1A) $$\int_{z}^{U}\left[KB(y)\Phi^{M-1}(y;z,T)-KB(z)\Phi^{M-1}(z;z,T)\right]f(y)dy-C=0$$

The only valid means of comparing the cost of the fixed-prize tournament with the auction-style tournament is to compare them when they both induce the same *z-stop* value. This means both contests will have the same distribution of final innovations, so the *ex ante* best tournament is the one with the lowest expected cost. The following lemma and theorem prove the expected cost of an auction-style tournament is always less than the prize required to generate an equivalent level of effort in the fixed-prize tournament.

Lemma 2.1: *For a fixed-prize tournament and an auction-style tournament with the same values of z-stops exceeding zero, P > KB(z).*

Proof: Assume *KB(z) ≥ P.*

From (1P) $$P\int_{z}^{U}\left[\Phi^{M-1}(y;z,T)-\Phi^{M-1}(z;z,T)\right]f(y)dy-C=0$$

Substituting *KB(z)* for P :

$$\int_{z}^{U}\left[KB(z)\Phi^{M-1}(y;z,T)-KB(z)\Phi^{M-1}(z;z,T)\right]f(y)dy-C\geq 0$$

$y > z \Rightarrow KB(y) > KB(z) \Rightarrow$

$$\int_{z}^{U}\left[KB(y)\Phi^{M-1}(y;z,T)-KB(z)\Phi^{M-1}(z;z,T)\right]f(y)dy-C>0$$

From (1A) $$\int_z^U \left[KB(y)\Phi^{M-1}(y;z,T) - KB(z)\Phi^{M-1}(z;z,T) \right] f(y)dy - C = 0$$

Contradiction. Therefore, $P > KB(z)$. **Q.E.D.**

Lemma 2.1 tells us the equilibrium prize payment, *KB(x),* for a firm which submits an innovation which is exactly equal to the *z-stop* is always strictly less than the size of the fixed-prize required to induce that same value of *z-stop*. However, we are interested in the total expected cost of the auction-style tournament compared to the prize of the equivalent fixed-prize tournament. The following theorem tells us that the auction-style tournament is cheaper.

THEOREM 2.1: *The expected cost of the auction-style tournament is strictly less than the prize awarded in a fixed-prize tournament with the same positive z-stop.*

Proof of Theorem 2.1: We can equate the left-hand sides of (1A) and (1P):

$$P\int_z^U \left[\Phi^{M-1}(y;z,T) - \Phi^{M-1}(z;z,T) \right] f(y)dy = \int_z^U \left[KB(y)\Phi^{M-1}(y;z,T) - KB(z)\Phi^{M-1}(z;z,T) \right] f(y)dy$$

Rearranging:

$$\int_z^U [P - KB(y)]\Phi^{M-1}(y;z,T)f(y)dy = \int_z^U [P - KB(z)]\Phi^{M-1}(z;z,T)f(y)dy$$

From Lemma 1, $P > KB(z) \Rightarrow \int_z^U [P - KB(y)]\Phi^{M-1}(y;z,T)f(y)dy > 0 \Rightarrow$

$$P\int_z^U \Phi^{M-1}(y;z,T)f(y)dy > \int_z^U KB(y)\Phi^{M-1}(y;z,T)f(y)dy$$

Multiply $f(y)$ on both sides by $\dfrac{1-F^T(z)}{1-F(z)}$ (becoming $\phi^U(y)$ because $y > z$) to get:

$$P\int_z^U \Phi^{M-1}(y;z,T)\phi(y)dy > \int_z^U KB(y)\Phi^{M-1}(y;z,T)\phi(y)dy$$

$P > KB(z) > KB(x)$, $\forall\, x < z \Rightarrow$

$$P\int_0^z \Phi^{M-1}(y;z,T)\phi(y)dy > \int_0^z KB(y)\Phi^{M-1}(y;z,T)\phi(y)dy$$

Add this inequality to the previous inequality and multiply everything by M to get:

$$P\int_0^U M\Phi^{M-1}(y;z,T)\phi(y)dy > \int_0^U KB(y)M\Phi^{M-1}(y;z,T)\phi(y)dy$$

Left side reduces to P and the right side is expected cost of auction-style tourney.

($M\Phi^{M-1}(y;z,T)\phi(y)$ is just the largest order statistic pdf of M draws.) **Q.E.D.**

Theorem 2.1 is a powerful result that says if fixed-prize tournament and an auction-style tournament have the same z-stops, the auction-style tournament

always has a smaller expected cost -- regardless of how large we needed to make *K* to achieve that *z-stop*. Therefore, even if we have to commit to paying the winning contestant 10 times his final bid to obtain some desired level of effort, an auction will still be cheaper than inducing this amount of effort with a fixed prize. Equivalently, one could say that for the same expected cost to the sponsor, the auction-style tournament always induces a greater level of effort by contestants.

To illustrate the performance of the auction-style tournament compared to the fixed-prize tournament, below I solved for the *z-stops* and expected costs when *F(x)* is uniform[0,100], M = 3, K = 1, and C = 1. (Numerical values and further examples can be found in Appendix 2B)

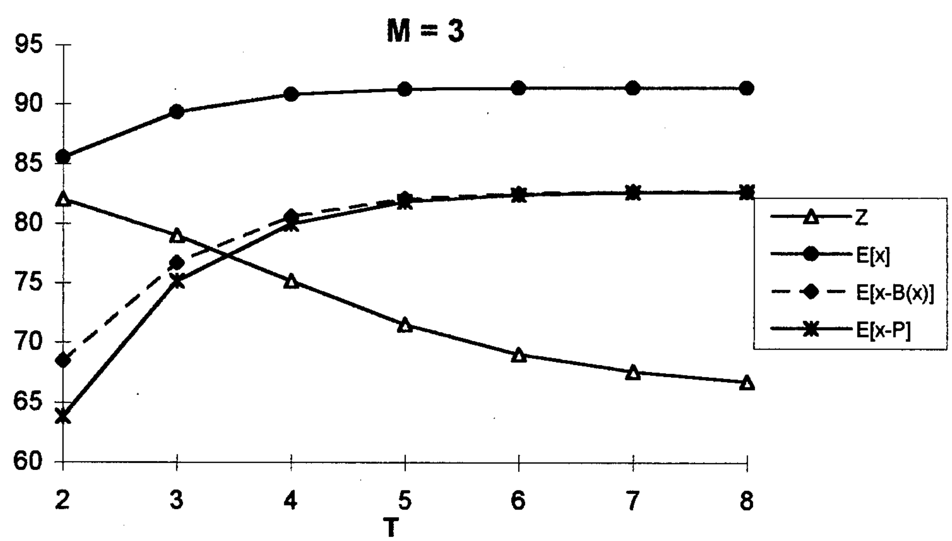

Figure 2.1

Figure 2.1 has only a single line plotting *E(x)* because both tournaments have the same *z*-stop so they both have the same *E(x).* Notice that as the length of the tournament increases and the expected surplus nears its peak, the difference between the auction-style tournament and the fixed-prize tournament becomes relatively small. This suggests that in lengthy tournaments the auction-style is probably more useful primarily because it reduces the burden of information on the sponsor. Conducting an auction-style tournament is easier on the sponsor and, on average, the sponsor will achieve a larger surplus than with the equivalent fixed-prize tournament.

To show that the sponsor gains more flexibility in manipulating the *z-stop* in the auction-style tournament, lets assume that M=2 and *F(x)* is still uniform[0,100]. By offering a prize of 100 -- the highest innovation value possible -- the sponsor can raise the equilibrium *z-stop* to a maximum of 89.733. Since 100 is the largest prize ever rationally offered by a sponsor (a bigger prize would make it impossible for the sponsor to make a positive surplus) 89.733 is the logical upper bound on the z-stop for the fixed-prize tournament in this example. However, if we are willing to accept the third-party equilibrium solution (i.e., the play of a weakly dominated strategy by the sponsor and bids of *B(x)* by firms) then by using an auction-style tournament and K = 2.9539 the sponsor can raise the z-stop all the way to 98.44 and still *guarantee* that she never takes a loss. With this value of *K*, the sponsor's surplus is still strictly

increasing throughout the range of x.[8] Moreover, the sponsor's expected net payment in this auction-style tournament is only 78.771. In order to induce the same z-stop of 98.44 with a fixed-prize would require an astronomical prize of 4141.41. So, the auction-style tournament clearly increases the sponsor's ability to efficiently manipulate effort if the sponsor is willing to play a weakly dominated strategy and that strategy is credible.

2.5: CONCLUSION

It is remarkable that the auction-style tournament so strongly dominates the basic fixed-prize tournament, especially when one realizes that *less* information is required of the sponsor to run the tournament. The auction-style tournament performs better because the contestants have to compete in prices at the end of the contest in the auction-style tournament, while this extra competition is absent from the fixed-prize competition. In particular, larger prizes are awarded in equilibrium to firms with better innovations, so firms commit to greater levels of effort in attempts to win the larger prizes.

This chapter has proven that auction-style research tournaments induce greater levels of effort by contestants than fixed-prize tournaments with the same

[8] The equilibrium bid for each x value is $.3333x$ for all innovations smaller than 98.44 and almost identically .3333x for innovations larger than 98.44, therefore K = 2.9539 does not upset the monotonicity of the term $x\text{-}KB(x)$.

expected cost. I have also shown that by using a third party, the sponsor can theoretically manipulate the equilibrium level of effort without giving up the cost-saving benefits of the auction-style tournament. Theorem 2.1 provides a strong argument for conducting research tournaments requiring contestants to submit their own bid prices at the end of the tournament. In fact, this is fairly typical of what we see in practice such as in government sponsored procurement contests. This paper shows those policies are theoretically sound, even if they are implemented primarily for convenience. The auction format is cheaper for the sponsor, requires less information, and induces greater levels of effort.

Chapter 3

Tournaments and Competition

in

Defense Acquisition Reform

In the last few years the Department of Defense has called for radical reforms in the way the military buys new weapons. A centerpiece of the new acquisition reforms is a renewed emphasis on competition, but Pentagon officials have been calling for more competition in defense procurement for three decades. Recent work in economics has shown tournament-style competitions promote simple and efficient research programs. This chapter describes the basic elements of the theory behind research tournaments and analyzes DOD practices in light of that theory. The analysis suggests current procurement practices deviate substantially from the requirements for efficient competitions. These deviations are predominantly self-imposed by a system that is overburdened with regulation, laden with conflicting incentives, and lacking a credible commitment by the government to competitive outcomes.

3.1: INTRODUCTION

The United States sharply reduced its spending on defense following the collapse of the Soviet Union and without question the part of the defense budget that has suffered the greatest cuts is military procurement.[1] In 1985 the DOD budgeted 29 new ships, 943 new aircraft, and 720 new tanks, but by 1995 we had reduced our annual purchases to six ships, 127 aircraft, and zero tanks.[2] After adjusting for inflation, the proposed 1996 procurement budget is 71 percent lower than the funding level reached in 1985 -- in real dollars, a fall of close to $90 billion -- and the administration's five year forecast projects procurement dollars to remain at reduced levels through the end of the century.[3]

Most of the adverse effects of procurement cutbacks have not been felt in the field primarily because the military has been rapidly downsizing and relying on an inventory of weapons it bought during the 1970s and 1980s. However, in recent testimony to Congress, Secretary of Defense William Perry acknowledged that the services must soon reverse the trend towards reduced procurement spending and begin replacing its aging weapons if they are to maintain their fighting edge into the

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[1] Defense spending in 1960 was equal to 8.2 percent of GDP and accounted for 45 percent of federal outlays. During the Vietnam War defense spending was typically between 7 and 8 percent of GDP and 35 to 40 percent of the federal budget. At the height of the Reagan buildup in 1985 American defense spending equaled 6.2 percent of GDP or roughly 26 percent of federal outlays. The proposed 1996 defense budget accounts for under 4 percent of GDP and about 15 percent of federal outlays. *Defense '94*, Issue 5, pg. 19 and Towell, 1995, pg. 461.

[2] News briefing by Secretary of Defense William Perry at the Pentagon Feb. 7, 1994. *Defense Issues, vol.* 9, no. 9. pg. 3.

[3] Towell, 1995, pg. 461,464

next decade.[4] This need to replace older equipment is driven not only by the equipments' deterioration, but also by changes in the world's military and political landscape. As Russia fell onto lean economic times in the 1990s it began selling its most technologically advanced military weapons to virtually any nation willing to pay in hard currency.[5] While the U.S. military no longer faces the same immediate threat of a massive armored attack in Europe, it now faces the prospect of fighting smaller armies equipped with extremely potent high-tech weapons in battles anywhere around the globe. This is a major defense problem because one of the clear lessons from the Persian Gulf experience is that even a small force equipped with advanced weapons can potentially inflict great damage on any Army.

Part of Secretary Perry's plan for revitalizing defense procurement rests on the ability of the Department of Defense (DOD) and Congress to radically reform the military's acquisition policies. By reforming acquisition the government hopes to save billions of dollars annually by reducing waste and lowering the costs of weapons purchases. Besides streamlining current procedures, a centerpiece of the new acquisition reforms being sought by the DOD is a renewed emphasis on competition and making acquisition procedures more compatible with commercial

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[4] Testimony to Senate Armed Services Committee on Feb. 9, 1995. Rpt by Towell, 1995, pg. 461.
[5] For example, Aviation Week and Space Technology has reported Russian sales of its high-tech SA-10 missile system as well as its most advanced MiG-29 and Su-27 fighters to China and other nations across Asia. Russian officials have also acknowledged that a stealthy Russian version of the Advanced Tactical Fighter is being developed and expected to enter service in the next 3-5 years. (Aviation Week & Space Technology, Jan. 2, 1995, pg. 60 and Sept. 12, 1994.)

practices to better take advantage of market forces.[6] But, Secretary Perry's desire to capitalize on the benefits of competition is not new. Pentagon officials and defense analysts have been calling for more competition in defense procurement, with sporadic success, for more than 30 years.

In 1965, then Secretary of Defense Robert S. McNamara testified to Congress that savings of 25 percent or more could be achieved by switching to competitive acquisitions.[7] In the decades since, Congress and the Department of Defense have repeatedly sought legislative reforms to make competition a central tenet of the acquisition program. Today, acquisition strategies must include provisions for competitive prototyping[8] and the designation of a senior-ranking "competition advocate" responsible for planning and promoting "full and open competition" in each phase of the acquisition cycle.[9] Recently, theoretical breakthroughs in economics have proven tournament-style competitions can constitute simple and efficient acquisition programs.[10] Presumably, it is this type of competitive environment which the DOD has repeatedly sought to capitalize on over

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[6] See the remarks of Secretary Perry in *Defense Issues,* vol. 8, no. 65.
[7] Commander Benjamin R. Sellers, "Second Sourcing: A Way to Enhance Production Competition," *Program Manager,* vol. 12 (May-June 1983), p. 12. (Rpt by Pilling, pg. 1).
[8] Competitive prototyping is required unless the program's decision authority approves a waiver and submits written notification to Congress demonstrating that competitive prototyping is not practicable. DODI 5000.2, Feb. 23, 1991 -- based on statutory requirements of Title 10 U.S. Code 2365, reference e.
[9] DODI 5000.2, Feb. 23, 1991, Part 5, Section A, pg. 3. This requirement is a result of Title 41, U.S. Code, Section 418, "Advocates for competition" and Title 10, U.S. Code, Section 2318, "Advocates for competition."
[10] For example, see Curtis Taylor, "Digging for Golden Carrots: An Analysis of Research Tournaments," *American Economic Review,* (forthcoming June, 1995).

the past three decades. This chapter describes the most basic points of the theory behind efficient research and development tournaments and analyzes DOD acquisition practices in light of that theory. The comparison suggests the current DOD procurement practices deviate substantially from the requirements for efficient competitions. These deviations are predominantly self-imposed by an acquisition system that is overburdened with excessive regulation, laden with conflicting objectives, and lacking an observable commitment by the government which reduces the incentives of firms to engage in effective competition.

3.2: THE NEED FOR CHANGE

Though the DOD has increasingly relied on different forms of competitive acquisitions in recent years, it is still plagued by allegations of widespread fraud and waste.[11] A senior Pentagon official estimated in 1989 that DOD wasted twenty to thirty cents of every dollar it spends on procurement.[12] Last year, Secretary of Defense William Perry cited a study which found the overhead, management and

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[11] Evidently, media accusations of *fraud* are usually overstated. Contrary to public opinion, the Packard Commission on Defense Management concluded that fraud is not an overwhelming problem in defense procurements. On the other hand, they concluded that inefficiency is a major problem noting, "The nation's defense programs lose far more to inefficient procedures than to fraud and dishonesty." *A Quest for Excellence*, 1986, pg. xxiii. Gansler, 1989, pg. 197 reports similar findings, suggesting that only about one dollar in 10,000 is lost to illegal activities in defense acquisitions.

[12] Remarks of Robert Costello. former under secretary of defense for acquisition. Rpt by Mendel and Stubbing, 1989, pg. 53.

control costs associated with DOD acquisition account for about 40 percent of the acquisition budget compared to only 10 to 15 percent for commercial firms.[13] So, despite calls for a greater emphasis on competition, the DOD has apparently still not seen the large gains in purchasing power predicted by its earliest advocates. However, this is not necessarily due to the failure of competition per se, but rather to the unusual, excessively regulated environment in which DOD competition is required to succeed.

As the DOD increased its use of competitive acquisitions, it failed to correspondingly decrease its regulatory requirements. By the mid-1980s, the Pentagon's book of procurement rules had grown to more than 7,500 pages, and contained an additional 30,000 pages of "policy guidance."[14] Former Navy Secretary John Lehman, Jr. lamented, "By actual measurement in 1985, existing legislation and case law governing navy procurement alone had grown to 1,152 linear feet of shelf space in the library."[15] Instead of addressing the entire acquisition process, past reforms were initiated to treat the various unpleasant symptoms of the procurement cycle, such as cost overruns or schedule slippages, rather than correcting the basic incentive problems which caused the disagreeable symptoms. Repeated attempts at reforming the procurement process over the years

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[13] The Carnegie Commission on Science, Technology and Government, "A Radical Reform of the Defense Acquisition System," Dec. 1, 1992. (Rpt in *Defense Issues* , vol. 9, no. 10, pg. 3)
[14] Jacques Gansler, 1989, pg. 151.
[15] John F. Lehman, Jr. Command of the Seas, (New York: Scribners, 1989) Rpt in William E. Kovacic, "The Sorcerer's Apprentice" pg. 105.

merely added to the confusing nature of the acquisition system, so today effective competition is virtually impossible to achieve. In the words of Secretary Perry, "While each rule individually has (or had) a purpose for its adoption and may be important to the process as a whole, it often adds no value to the product itself and when combined, contributes to an overloaded system that is often paralyzed and ineffectual, and at best cumbersome and complex."[16] In 1986, the Packard Commission (Presidential Blue Ribbon Commission on Defense Management) charged, "Federal law governing procurement has become overwhelmingly complex. . . . As law and regulation have proliferated, defense acquisition has become ever more bureaucratic and encumbered by unproductive layers of management and overstaffing."[17] To reduce bureaucratic oversight, one recommendation by the Packard Commission was for "substantially increased use of commercial-style competition, relying on inherent market forces instead of government intervention."[18]

The economic marvel of "the market" is not so much that it is efficient, but rather that it requires so little information to be efficient. This point was emphasized by Hayek who observed that the price system dispenses with "the need of conscious control and how to provide inducements which will make the individuals do the

[16] Testimony provided to the House Armed Services Committee, Feb. 9, 1994. Rpt in *Defense Issues, vol. 9, no. 10*, pg. 4.
[17] *A Quest for Excellence*, 1986, pg. xxii
[18] *A Quest for Excellence*, 1986, pg. xxvi

desirable things without anyone having to tell them what to do."[19] This is not to suggest that the defense market closely approximates a theoretical free-market environment, but only to point out that by combining layers of confusing regulatory requirements on top of the competition, we create expensive informational barriers that restrict the usefulness of competition regardless of the number of buyers or sellers. Neither the competition nor the regulations are as effective as they could be in a simpler, more coherent policy environment. Overly detailed mil-spec requirements take away the ability of contractors to compete with innovative designs that improve performance and lower costs. Repeatedly "bailing out" struggling contractors and renegotiating contracts on a "fair profit" basis creates disincentives for truth telling and raises the probability that inefficient firms will be awarded production contracts. Annual line-item budget reviews by Congress eliminate the ability of the DOD to offer credible prizes.

While there is much room for improving the current acquisition system, inappropriate changes could also make procurement programs much worse.[20] The

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[19] Hayek, F. A., "The Uses of Knowledge in Society," *American Economic Review*, 35, Sept. 1945, pg. 527 (Rpt Smith, 1982.)

[20] Frederick Biery analyzed 35 different kinds of large, high-technology undertakings representing over 200 distinct projects from both commercial and public enterprises such as highways, satellites, electronics, construction projects and pharmaceutical drugs development. His data show that on average these projects experienced a cost overrun of 96 percent. However, the average cost overrun for a set of 55 military weapon systems was 47 percent – roughly half that of other similar projects. Biery also looked at schedule slippages and found that military systems had an average increase in schedule growth of 33 percent while commercial projects had an average of 84 percent schedule overrun. See Biery, 1992: 637-664.

biggest need in acquisition reform is simplification and a better focus on the proper incentives to let competition work. Fortunately, efficient research tournaments are very simple programs which should not require an excessive amount of regulation and oversight. If the government values competition, it should eliminate the regulations and oversight that interfere with competition and credibly commit to the use of effective competition or it cannot expect to succeed. On the other hand, if the government is unable to commit to competitive outcomes, or if competition is not cost-effective in some phases of the acquisition cycle, there is no reason to retain the facade of competition because it merely clouds the decision making process.

3.3: ECONOMICS AND THE DEFENSE MARKET

The defense industry is clearly not the textbook competitive free-market that we think of in economic theory.[21] The number of companies allowed to compete as prime contractors for any new major weapon system is usually limited because of the enormous investment costs involved in developing these weapons. For example, through 1991, the public cost of developing the Advanced Tactical Fighter which had not yet entered production had risen to $3.9 billion and B-2 bomber

Besides the study by Biery, Jacques Gansler notes similar findings by the General Accounting Office, Report B163058 July 1972, and the House Committee on Government Operations, Nov. 16, 1979. See Gansler, 1989, pg. 4.

[21] Gansler lists two pages of differences between free-market theory and the defense market. Gansler, 1989, pg. 159,160.

development already had a price tag of $33 billion.[22] So, by the time a major weapon system is ready to enter production there is generally only one, or on rare occasions two, potential producers. Additionally, the U.S. government is the only legal consumer for most new military weapons. Therefore, at the time final purchases are negotiated the market is most accurately characterized as a bilateral monopoly.

Bilateral monopolies are known to be problematic in economic theory.[23] First, they tend to induce less than the efficient amounts of investment. Defense firms may be unwilling to invest in research and development because they face substantial losses if Congress or the military act opportunistically and exercise their monopsonistic powers at the negotiating table after the investments are made. This economic problem of "hold-up" is a direct result of the government's inability to make credible commitments to contractors because of its annual budget review process. Ben Rich, former head of Lockheed's Skunk Works, laments, "Frankly, I think the government prefers this annual funding system because it can then promise a company the moon and the stars in order to get it to put up significant development capital, then later sharply reduce the procurement, as in the case of the F-22."[24] On the other hand, if the government opts ahead of time to reimburse firms

[22] Easterbrook, 1991, pg. 51.
[23] For a simple but descriptive discussion of problems associated with bilateral monopolies see Tirole, 1989, pg. 21-26.
[24] Janos and Rich. 1994, pg. 335.

for their investments, firms have little incentive to reduce waste and invest in the appropriate types of research and capital. In economic jargon, this problem is known as moral hazard.

Even if the efficient amount of investment is undertaken, there are still substantial problems facing a bilateral monopoly due to differences in information known to the buyer and the seller. For example, Myerson and Satterthwaite (1983) have proven that as long as the buyer's value and the seller's costs are private information, there exists no efficient bargaining process because each party has an incentive to "hedge" the value or cost it reports to the other agent. There are also a broad range of other economic problems inherent in defense contracting which may not be found in other procurement environments. The U.S. government is the buyer, but the primary user (the military) is not the agent that pays the bills (Congress). This creates problems because Congress and the military are motivated differently, and each has different information. The government may also feel that it faces limitations on the amount of competition it can impose on the defense industry without causing some important firms to exit. Both Congress and the military have an interest in maintaining a surge capability for weapons production in times of national emergency, and Congress in particular has a political motivation to keep defense jobs and contracts flowing into individual congressional districts.[25] While

[25] For example, the subcontracts for the B-1 bomber created 140,000 jobs and are distributed across over 400 congressional districts in every state except Alaska and Hawaii. Gansler, 1989, pg. 85.

this interest in preserving a viable defense industry helps to mitigate the effects of hold up, it also damages attempts at creating impartial competition among defense firms.

Finally, enormous risks and uncertainties permeate the weapons acquisitions process. It is not unusual for new major weapons systems to take more than a decade to go from conceptualization to deployment. During this lengthy period of time there are uncertainties about the success of new technologies, uncertainties about future rates of inflation and the costs of raw materials, uncertainties about the evolving nature of perceived military threats which the weapon was conceived to deter, and there are uncertainties about foreign and domestic politics -- all of which make weapons development and production very risky endeavors for industry and government alike.

3.4: RESEARCH TOURNAMENTS IN ECONOMIC THEORY

Contracting directly for research and development is extremely troublesome because of the massive uncertainties involved in discovering and applying new technologies and the difficulty of accurately observing contractor efforts and expenses. Because of these difficulties, William Rogerson[26] has argued that the

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[26] Rogerson, *JPE, 1989.*

DOD must give firms an economic incentive to exert effort by promising to reward successful innovations with "prizes." A regulatory structure that directly rewards firms with larger prizes for higher-quality innovations is desirable, however Rogerson suggests it is generally not feasible because the value of weapons innovations cannot be objectively judged and verified by the courts and therefore any prize awarded would always be subject to dispute by at least one party.

To solve these problems, Taylor has suggested creating a research tournament.[27] In Taylor's research tournament model, firms compete for a predetermined prize offered by the tournament sponsor (e.g., the government). The way Taylor's tournament works is as follows: at the beginning of the tournament, the government announces the prize to be awarded to the firm producing the best innovation at some fixed date in the future corresponding to the end of the research tournament. Along with the prize commitment, the government also specifies its innovation preferences so that firms understand which kinds of innovations to pursue. If there are a large number of firms wishing to compete for the prize, the government must limit the number of firms allowed to compete in the tournament in order to increase the amount of research effort put forth by each firm. Since only a few firms are allowed to compete, each firm has a strictly positive expected profit from the tournament. Therefore, to extract the excess expected profits of each firm

[27] Taylor, 1995.

the government can charge all entrants a fee to compete. In theory, charging the entry fee will not affect the firms' research behavior because it is a "sunk" cost once the competition begins. After the entrants pay the entry fee, they conduct research as desired until the specified date when innovations are judged. On that final date, each firm turns over its best innovation to be judged and the government evaluates each innovation then selects one "winner." The firm producing the winning innovation is given the prize, and the government gets the innovation.

A research tournament differs from a patent race because the tournament sponsor always awards the prize on a predetermined *date* to whichever firm has the best innovation at that particular time. In contrast, a patent race is open ended with the winner being the first firm to produce a prespecified *innovation*, independent of the time or date.[28] Verifiability is not a problem in this research tournament, because the courts need only observe whether or not the predetermined prize is actually paid at the designated time. Since prize payment is easily verifiable by a court, firms are willing to invest in costly research and development efforts to try and win the prize without fearing that the government will renege on its contract after the investments are made. By choosing a sufficiently large prize, and limiting the tournament to a few well-qualified firms, the government is able to induce the desired amount of competitive effort by each contestant.

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[28] For a more thorough discussion and examples of the difference between a research tournament and a patent race see Taylor, 1995.

One of the key requirements in an effective research tournament is that the sponsor must restrict entry to a limited number of competitors.[29] Without entry restrictions, as the number of competitors grows larger each firm's chances of winning begins to shrink so they put forth less effort and expend fewer resources to develop the new innovation. This requirement to limit the number of competitors sharply contrasts with the statutory provisions of the Competition in Contracting Act (CICA) of 1984 which requires the DOD to conduct "free and open competition" for its procurement contracts.[30]

If there are many firms interested in competing for the prize, the government must invite only a few to compete or each entrant's chance of winning the prize will be so small that they will not conduct enough research to be effective. The government can restrict entry and simultaneously reduce costs by charging a fee to all firms "invited" to compete. However, one of the problems with Taylor's tournament is how the government can figure out the appropriate size of the entry fee. If the entry fee is set too high, firms will refuse to enter the tournament. If the fee is set too low, the firms will enter but the government will be leaving money "on the table." The government may be tempted to add to the competition by holding an

[29] Taylor, 1995, Proposition 3.1.

[30] In the CICA of 1984 (Public Law 98-92) Congress removed the previous requirement for "effective competition" and now requires "free and open competition" in the interest of "fairness" to all sellers. Unfortunately, now the DOD often spends far too much money evaluating bids, independent of the effect on competition. For example, Gansler cites an Army competition to purchase an $11,000 item but after advertising the contract the Army got over 100 responses and spent over $5000 just reproducing and sending out the invitations. Gansler, 1989, pg. 182.

entry auction prior to the contest and having firms bid to gain entry in the tournament. While this mechanism seems like a practical method for restricting entry, it may not be optimal in all situations. In chapter one I have shown that entry auctions are efficient only when firms differ in very specific ways. If firms differ only in their costs of research effort, then the firms willing to bid the most to enter the tournament will correspond to the firms with the lowest research costs. This analogy is easy to understand, because it just means that firms with a cost advantage have a higher expected profit and therefore value entry more. On the other hand, if firms start with different innovations, I have also shown that conducting an auction may not be efficient for selecting firms. The firms that start out with the best innovations may not bid high enough to guarantee entry in the tournament. Thus, conducting a simple entry auction may not always result in the best firms gaining entry. Fortunately, in the defense acquisition business there are relatively few firms qualified to be prime contractors and the government has a reasonable amount of information about the abilities of each. Thus, restricting entry may not be an insurmountable problem for the government on major defense contracts, but it may require amending the current Competition in Contracting Act. Also, since the DOD typically conducts a substantial amount of its own weapons research and shares this information with major contractors, it is reasonable to assume that most of the prime contractors start out on reasonably equal footing so differences in starting

technologies may have less of an impact on entry decisions compared to other factors.

One may complain that charging entry fees is not a valid concept in defense procurement because the DOD generally pays firms to compete by awarding early development contracts and sharing R&D costs. I believe this concern is overstated, for example Northrop developed the F-20 fighter on its own, using private funds. As a better example, consider Lockheed and its highly acclaimed F-117 stealth fighter. At the time the DOD initiated the competition for what was to become the F-117, the Air Force gave Northrop, McDonnell Douglas, and three other companies a million dollars each to come up with a conceptual design for the stealthy aircraft -- but Lockheed wasn't even invited to participate. However, the people at Lockheed's Skunk Work facilities knew they had a winning design concept, so they went to the Air Force and asked to join the competition at no cost to the government.[31] Once they won the design phase, the Air Force paid them $20 million to purchase two prototypes but Lockheed needed $30 million so again they privately funded the extra $10 million to build the two aircraft.[32] Indeed, I think a case can be made that if firms believe in their capabilities and the government is willing to *commit* to awarding a worthwhile prize, firms will be willing to invest their own funds to win the contest. The primary reason that firms are leery of

[31] Janos and Rich, 1994, pg. 23.
[32] Janos and Rich, 1994, pg. 40.

investing their own funds today is precisely because there is no tangible commitment by the government to awarding a prize, as will be shown later in the paper.

One problem Rogerson emphasized that is not solved by Taylor's research tournament is the ability to award larger prizes to firms that develop better innovations. Under Taylor's fixed-prize tournament, the winner is awarded the same prize regardless of the quality of its winning innovation. Additionally, the fixed-prize tournament requires the government to figure out some method of determining the proper size of the prize to induce efficient levels of effort. It is highly unlikely the government will ever know enough about all the risks of emerging technologies and individual firm capabilities to accurately compute the efficient prize, so the government faces serious informational barriers if it ever desires to actually implement a Taylor-style research tournament. Optimally, we would like to reduce the government's information burden while simultaneously incorporating a prize structure that awards firms with larger prizes for better innovations in the hope that such a reward schedule will induce firms to work harder to develop better innovations. I have shown in chapter two that this is feasible if the government conducts an "auction" at the end of the tournament. In this auction-style tournament, at the end of the contest firms submit not only their winning innovations for the government to evaluate, but they also simultaneously submit bids or prices which they would charge the government for purchasing their innovation.

In this way, firms with lesser quality innovations can make their products more competitive by submitting lower bids, and firms with high-quality innovations can submit bids which charge a premium for better quality. Moreover, in equilibrium, firms which create the best innovations should earn the largest profits, making firms even more willing to invest private funds and continue researching to create better innovations. This means the auction-style tournament will produce better innovations for a lower expected cost than the fixed-prize tournament. The new auction-style tournament requires less information be known by the government, yet it induces greater effort by firms for the same expected cost.

In short, the four key ingredients of an efficient and implementable research tournament are:

1) The government *commits* ahead of time to paying out a "prize" that is verifiable by the courts so firms are willing to make costly investments to win.
2) The government *specifies* its preferences so firms know which innovations to pursue in order to have the best opportunity to win the prize.
3) The government *restricts* entry to only a few firms to raise each firm's chance of winning so the firms compete vigorously to win the prize.
4) Each firm submits their innovation along with a bid at the end of the tournament, and the government "purchases" whichever innovation/bid combination it believes offers the best value.

Note the simplicity of this tournament in contrast to the complex regulatory environment of the current system of competition. The tournament does not require extensive monitoring of costs and effort by the government. All the government must do is make a legally binding commitment to reward the winner, limit the number of competitors, and pick a winner at the end of the tournament. If the government desires more flexibility in determining the tournament prize or wishes to set upper and lower bounds on the size of the final bids, it can simply stipulate those limits prior to the beginning of the tournament so firms will adjust their research efforts and their bids appropriately Also, the government has the option of having an impartial court hold each firm's sealed bid while the government independently evaluates the innovations, thereby providing some assurance that the government is not practicing overt favoritism in the awarding of contracts.

3.5: PHASES OF THE DEFENSE ACQUISITION CYCLE

The defense acquisition cycle is divided into five phases: concept exploration and definition, demonstration and validation, engineering and manufacturing development, production and deployment, and finally operations and support.[33] Each of these phases is separated by milestone decision points which

[33] DOD regulation 5000.2, February 1991.

require the contractor to demonstrate success with the proposed system in the previous phase prior to moving to the next phase of the cycle.

Following the determination of need for a new weapon, in the concept exploration phase, the services investigate various alternatives to satisfying the military's need and attempt to define the most promising weapon concept. During this phase the DOD completes a system threat assessment and establishes key system characteristics and operational constraints. The DOD also makes initial assessments of the primary risks associated with the concept, and forms a preliminary acquisition strategy with program objectives for cost, schedule, and performance.

In the demonstration and validation phase, contracts are awarded for developing multiple design approaches and parallel technologies that satisfy the system concept. The objectives of this phase are to better define the important design characteristics and expected capabilities of the system concept, demonstrate that critical technologies can be incorporated into the system with confidence, and establish a proposed development baseline containing refined program cost, schedule and performance objectives for the most promising design. To aid in the decision making process during this phase contractors may be required to produce testable prototypes of various system components to provide the DOD with reasonable assurance that the concept is both feasible and affordable.

According to DOD Instruction 5000.2, in the engineering and manufacturing development phase, the most promising approach developed in the demonstration and validation phase is translated into a "stable, producible and cost effective system design."[34] At this time, extensive use of prototypes for operational testing and evaluation of the system and the manufacturing process occur to provide a realistic idea of performance under operational conditions and realistic estimates of lifetime production costs as well as annual funding requirements. Sometimes, to save money and time to deployment, a system will enter low-rate "concurrent" production in this phase while development is still ongoing. The primary risk of concurrency is that the services may end up with a number of weapons which require extensive modifications at a later date to correct design deficiencies. For example, concurrent production was used in the 1980s with the B-1B but the program experienced significant post-production problems on the earliest production aircraft with avionics failures and fuel leaks.[35] In contrast, concurrent production was planned and incorporated successfully in the acquisition of the F-117 stealth fighter.[36] Finally,

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[34] DOD Instruction 5000.2, pg. 3-21.

[35] Kenneth Mayer mentions a GAO study that found as of March 1988, approximately 40 percent of B-1s had fuel leaks and 10 percent of the fleet had leaks severe enough to require grounding. General Accounting Office, *B1-B Cost and Performance Remain* Uncertain (GAO/NSIAD-89-55), Feb 1989. Mayer, 1991, pg. 56,57.

[36] Using the F-117 as an example, Ben Rich advocates planned concurrency for future airplanes as being cost effective when firms keep detailed parts records on all production models and design easy access to all onboard avionics and flight control systems. Rich and Janos, 1994 , pg. 332.

after all the bugs are worked out, the system enters high-rate production and is deployed in the field with the respective military services.[37]

3.6: ACQUISITION COMPETITION IN PRACTICE

Most often the public thinks of procurement competition as a sealed-bid auction that awards the contract to the lowest bidder, but competition can take on many different forms and occur at several different stages of the acquisition cycle. DOD regulations call for acquisition strategies which include provisions for competitive prototyping and competitive alternative sources from the beginning of development through the end of procurement.[38] Competitions in the earliest phases of the acquisition cycle usually center on basic design concepts with "paper" proposals and bids. For example, the Pentagon recently awarded 24 contracts totaling $127.8 million to 18 U.S. companies for the concept definition and design phase of the Joint Advanced Strike Technology .program (JAST).[39] During

[37] Actually, contract and hardware changes continue to be made long after the weapon is produced and deployed. As an example, Jacques Gansler points to the F-111 aircraft which had a total of 394,922 changes made during the life of the program. Gansler, 1989, pg. 166.
[38] DODI 5000.2 requires competitive prototyping unless the milestone decision authority (generally the under secretary of defense for acquisition or the DOD Component Head) approves a waiver and submits written notification to Congress that competitive prototyping is not practicable. 5000.2 also requires the option for competitive alternative sources throughout the period from the beginning of full scale engineering development through the end of procurement. See DODI 5000.2 pg. 3-9.
[39] Morrocco, 1995, pg. 22.

intermediate phases of the cycle, competition may also include the use of prototypes where firms actually demonstrate their ability to meet design criteria. The latest big-ticket prototype competition occurred in 1991 when the Air Force conducted a "fly-off" competition between Lockheed and Northrop for the right to build the stealthy Advanced Tactical Fighter (ATF). Lockheed's YF-22 prototype won the contract which at the time was expected to be worth more than $90 billion over the extended life of the contract.[40] Finally, there may be production competitions at the end of the acquisition cycle where firms compete for the right to manufacture the winning weapon design. For example, Hughes triumphed over Raytheon in a 1981 development competition, winning the contract to produce the Advanced Medium Range Air-to-Air Missile (AMRAAM), however, by 1985 Raytheon was reentered as a competitive second production source and subsequently awarded a contract to build 115 of the 260 missiles planned for fiscal 1987.[41]

The implications of competition differ across varying stages of the procurement cycle because of new information garnered at each stage and because it takes so long for major weapon systems to make their way through the procurement process. So, while we often speak in broad terms about the need to increase

[40] It is estimated that each company spent close to $1 billion in private and public funds on the development of each prototype. See Schwartz et al., 1991, pg. 46, 47.
[41] "Raytheon Receives USAF Contract as Second AMRAAM Producer" *Aviation Week and Science Technology*, Nov. 11, 1985, p. 20. (Rpt Burnett, 1987, p. 33.)

competition in defense acquisition, the effectiveness of competition depends on the form and the timing of the contest.

3.6A: Prototype Competitions during Development

In the late 1960s, then Secretary of Defense David Packard instituted the original "fly-before-buy" acquisition strategy which emphasized hardware testing and validation prior to beginning production though it was not entirely embraced by the military services.[42] In 1986 David Packard, chair of the Packard Commission, once again called for the DOD to institute prototype building in the acquisition system, recommending,[43]

> A high priority should be given to building and testing prototype systems and subsystems before proceeding with full-scale development. This early phase of R&D should employ extensive informal competition and use streamlined procurement processes. . . This increased emphasis on prototyping should allow us to "fly and know how much it will cost before we buy."

Today prototype building and testing is required in the Demonstration and Validation phase for all acquisition category I programs.[44] DOD Instruction 5000.2

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[42] Drezner, 1992, pg. 1.
[43] A Quest for Excellence, 1986, pg. xxvi
[44] Category I programs are those that are estimated to require an eventual expenditure of more than $200 million for research, development, testing, and evaluation, or if it is expected to eventually require more than $1 billion for system procurement. (Dollars are expressed in constant FY 1980 amounts). Competitive prototyping is required in accordance with Title 10, U.S. Code, Section 2365 unless a waiver is approved by the milestone decision authority and Congress is notified in writing. See DODI 5000.2, Feb. 91, section 2-3, section 3 and section 5-D

also suggests that selected prototyping may continue into the Engineering and Manufacturing Development stage as necessary "to identify and resolve specific design and manufacturing risks. . ."[45]

According to Drezner the perceived benefits of building prototypes include reducing technological risk and uncertainty, enabling better decisions concerning the trade-offs between cost, schedule, and performance, identifying system interfaces and key technical problems, increasing design options, and earlier development and operational testing.[46] Another important benefit of prototyping is that it is generally embraced by weapons developers. Ben Rich of Lockheed has written,

> My years inside the Skunk Works, for example, convinced me of the tremendous value of building prototypes. I am a true believer. The beauty of a prototype is that it can be evaluated and its uses clarified before costly investments for large numbers are made. Prior to purchasing a fleet of new billion-dollar bombers, the Air Force can intensively audition four or five, learn how to use them most effectively on different kinds of missions and how to maximize new technologies on board. They can also discover how to best combine the new bombers with others in the inventory to achieve maximum combat effectiveness.

Prototypes can be built and tested during any stage of the acquisition cycle. In Drezner's survey of 31 prototyping programs, 45 percent of the prototypes were

[45] DODI 5000.2, Feb. 91, section 3 and section 5-D
[46] Drezner, pg. 8, Table 2.1.

constructed during full-scale development and 35 percent were built during the demonstration and validation phase.[47] Unfortunately, Drezner was unable to find any statistical relationship between the use of prototypes and program improvements because when prototyping is done he could only guess at what outcome would have occurred if there had not been any prototyping.[48] (This is basically the same problem that is faced when trying to determine the cost savings from second-sourcing versus sole-sourcing.) However, I believe one of the main reasons past prototyping has not been associated with more procurement successes is because it has often been conducted *after* the contract was awarded to a single winning firm. Frequently, the prototypes were built only as a tool for testing and validating a design by the winning firm and not as part of the formal competition. But, the primary benefits of competition are most likely to occur through the construction and testing of prototypes because that process provides the most useful information for discerning cost and performance trade-offs in the acquisition cycle. If competitions are held prior to getting that information, too many uncertainties will remain in the weapon's procurement to allow the DOD to pick the best weapon because the firms themselves will not even have enough information to make valid assessments of their production costs or performance capabilities.

[47] Drezner, pg. 43.
[48] Drezner, pg. 97,

The biggest argument against funding multiple prototypes in the development phase is due to the expense of the hardware. However, Gansler has noted that on average only 15 percent of the total costs of procuring a new weapon system are spent prior to production (almost zero in concept exploration, 3 percent in demonstration and validation, 12 percent in full-scale development). A full 85 percent of the total costs accrue during production, operations and support.[49] Yet as Gansler points out, the importance of funding competitive development prototypes is that it is in this phase that production and operation costs are "designed in" to the weapon system.[50] Therefore, even if the DOD must pay more to fund a competitor in full-scale development, the government should save enough in later stages of production and operations to make up for the extra costs up front. In fact, Easterbrook believes the full-scale prototype competition kept the price of the new F-22 program under control, while the lack of an adequate prototype competition contributed to the enormous cost of the B-2.[51] Additionally, by designing an acquisition program that properly incorporates prototype competitions, much of the unnecessary sole-source regulatory oversight can be eliminated, saving administration costs. However, if prototypes are put off until after a production contract is awarded, the winning firm has fewer incentives to make a cost-saving

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[49] Gansler, 1989, pg. 157.
[50] Gansler, 1989, pg. 185.
[51] Easterbrook, 1991, pg. 50-51.

production design and the government will be forced to rely more fully on oversight to ensure contractual performance.

By funding complete prototype competitions, the services have an opportunity to actually see and test what they are paying for, and enough information should be known by the end of prototype testing to enable Congress and the DOD to comfortably *commit* to sole-source production without need for subsequent production competition. In a review of procurement competition, Jim Leitzel emphasized the value of information gained from prototypes, writing: [52]

> Prototype competitions for major weapons systems are one route to improving information. By basing the initial award on completed prototypes, as opposed to paper competitions, much more information concerning the quality and cost of the systems is publicly available. . . . Furthermore, the number of late changes in the weapon specifications, a current source of lucrative contract renegotiations, can be reduced.

In the event the winning firm fails to perform in later stages, the government already has a second source with a viable substitute that it can call on as a ready competitor without the need to transfer technology and product secrets from the previous winner. From an economic perspective, prototype "fly-offs" appear to most closely approximate the necessary conditions of an efficient economic tournament, offering

[52] Leitzel, 1992, pg 51,52.

the government the best opportunity to exploit competition while retaining the profitable incentives for firms to invest in innovative research and development.

3.6B: Production Competition: Dual and Second-Sourcing

While the DOD has traditionally relied on competition during a weapon's development, periodically it has revived interest in production competition through policies that require two or more firms to compete to produce the winning weapon design. Dual sourcing was widely publicized when the Air Force split the production contract between General Electric and Pratt & Whitney for building the high performance jet engines for the F-15 and F-16 fighter aircraft.[53] A few of the other major weapon systems that have been dual or second-sourced include the Shrike anti-radar missile, the Rockeye cluster bomb, the AIM-7F radar-guided missile, and the AMRAAM. By law, the DOD "requires that the acquisition strategy have an option for establishing competitive alternative sources for acquisition . . . if the results . . . show that the establishment and maintenance of two or more sources would likely reduce the technological risks associated with the program; likely result in reduced costs for such program; or likely result in an improvement in design commensurate with the additional cost. . . . (and) would not

[53] Drewes, 1987.

result in unacceptable delays . . . (or) is otherwise in the national security interests of the United States."[54]

The earliest empirical studies of procurement costs seemed to support the notion that second-sourcing and dual-sourcing in production leads to lower prices. For example, the Shrike anti-radar missile system sold at one time during sole-source procurement for $19,500 each, but after competition was introduced, the first producer's price dropped to $4,480 and the second source's price fell to $3,850.[55] Taken at face value, such evidence appears to strongly support the use of production competition to save costs. However, more recent analyses have uncovered important flaws in the empirical methods of those early studies.[56] Reexamining the evidence, Donald Pilling asserts: [57]

> The belief that competition saves money is based on comparisons of hypothetical sole-source prices with actual prices obtained under competition. . . Both the data and the analysis of the most frequently cited competition studies are questionable; the studies generally fail to account for the total costs of competition; and their models ignore the most relevant business strategies of the competitors.

[54] Title 10, United States Code, Section 2438, "Major programs; competitive alternate sources" as implemented by DOD Instruction 5000.2M pg. 4-D-2-2.
[55] Gansler, 1989, pg. 187.
[56] For a survey of empirical studies on second sourcing see Anton, J. and D. Yao, 1990.
[57] Donald Pilling, 1989, pg. 2.

Today, it appears that evidence on the impact of second-source production competition for reducing the total cost of weapons procurement is inconclusive at best, and more than likely production competition has substantially *raised* the total cost of procurement to the DOD for some weapons systems. For example, in a case study of the AIM-7F radar-guided missile, Michael Beltramo found that dual sourcing raised the lifetime cost of the program by between $165.3 and $189.7 million dollars (an increase of between 15.4 to 19.8 percent).[58]

The primary problem in studies empirically comparing sole-source production with competitive production is that these environments invoke different strategic behaviors by firms. Just because one finds that production prices drop when production competition is introduced does not necessarily mean the competition led to lower overall costs. Instead, the price drop may simply reflect earlier strategic behavior by firms, and an acknowledgment that firms have control over prices despite government oversight. Rational firms would anticipate the impending dual-sourcing and raise their sole-source prices prior to the start of competition. Figure 3.1 illustrates the evidence typically presented by analysts promoting the use of dual-sourcing to reduce costs.[59]

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[58] Beltramo, 1985, pg. 32.
[59] Gansler, 1989, pg. 187.

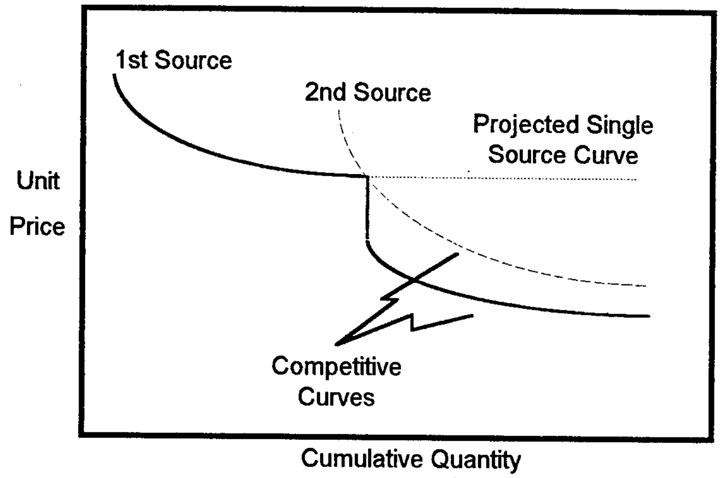

Figure 3.1

But, because initial sole-source pricing is affected by the strategic behavior of firms anticipating production competition, the price curve shown above is not the same curve that would have been elicited if the government had credibly committed exclusively to sole-source procurement. With a credible commitment, we can imagine the true sole-source pricing curve overlaid on the one shown above to get:

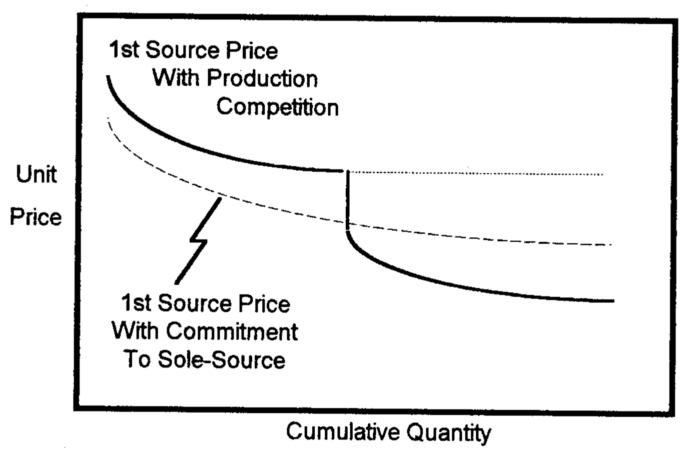

Figure 3.2

In theory, if firms anticipate large profits from production of a weapon these profits will largely be bid or "competed" away in development competition prior to the production phase. But if firms anticipate that the government will opportunistically impose dual sourcing to reduce production profits, then the net effect will be to reduce the level of competition during development as a buffer against later losses in production. This idea was expressed by Jim Leitzel writing:[60]

> If the competition between firms for the initial production award is intense, additional competition from a second source for later production contracts will simply result in less intense presource selection competition. If, alternatively, the initial award is largely non-competitive, there is little reason to suspect that the later production contract awards will suddenly elicit competitive behavior.

An important concept to understand regarding any competition is whether or not the government can make credible commitments to firms. If the government cannot make a credible commitment to avoid production competition, then firms may strategically raise prices in anticipation of later competition -- even when the DOD does not bring in another producer. Thus, by vacillating between policies the DOD probably gets a worse outcome than by either committing to sole-source production or always requiring competitive production. Unfortunately, this inability

[60] Leitzel, 1992, pg. 44.

to commit seems to reasonably describe our previous track record and is a critical departure from the necessary conditions for an efficient research tournament.

Beyond the unit costs of individual weapon systems as they roll off the production line, there are also substantial fixed costs associated with competitive production which must be overcome before the government ever breaks even. The government must bear the cost of transferring the new weapon's design technology to a rival firm and setting up an additional production line prior to invoking production competition. Another "hidden" cost of dual sourcing is the inefficiency it causes by splitting the total production buy which limits the effects of the "learning curve" to drive down production costs. Analyzing second-sourcing in a theoretical framework, Riordan and Sappington concluded, "second sourcing will often be undesirable, except possibly in special cases when the technology-transfer cost is of intermediate magnitude. Even then, sole sourcing will be preferred to second sourcing if the production-enhancement effect is weak."[61] For example, the Navy spent $237.9 million just to qualify a second source to produce the Phoenix missile. Thus it had to save more than that in unit cost reductions on subsequent purchases just to break even with the cost of qualifying the second source -- a prospect which Navy officials doubted could be done.[62] Therefore, production competition can only be cost-effective if the DOD expects the weapon system to be

[61] Riordan and Sappington, 1989, pg. 56.
[62] Pilling, 1989, pg. 24.

demanded in sufficiently large numbers to justify the expense of technology transfer and constructing an additional production line. This is why dual-sourcing is most often used for weapons like bombs and missiles where the government expects to purchase the system in large quantities.

There are also other important aspects in which introducing competition in the production phase may help or hinder DOD goals. On one hand, the mere *threat* of production competition may be useful for compelling a contractor to perform or reduce its price. Ernest Kish, planning officer for the High Speed Antiradiation Missile (HARM) Program Office, boasted in 1986, "With the threat of competition in the initial stages of production (program cancellation and dual sources) costs were stabilized early in the program."[63] Of course the problem with threats is that while the Navy may have saved money on the cost of the HARM missile by "holding up" Texas Instruments after it had already made substantial investments in the HARM missile, it is difficult to tell what kind of long-term effects this outcome had on the future behavior and bidding by Texas Instruments and other defense contractors. Ben Rich rather bluntly called the leader-follower competition instituted by the Navy "an absolute outrage and a debacle." suggesting that the

[63] In late 1982 and early 1983 the Navy wanted to develop a second-source for the High Speed Antiradiation Missile (HARM) produced by Texas Instruments, but was denied because the Defense System Acquisition Research council doubted the services could recoup the estimated $80 million cost of qualifying a second production source. However, during the intervening time the program management office entered negotiations with TI to reduce unit costs. Notably, the Air Force supported sole-source production. Kish, 1986, pg. 9-11.

services created a substantial amount of antagonism with the practice which may have cost the services more in the long-run as contractors reacted strategically to the additional competition.[64]

Independent of price effects, production competition may occasionally be relied upon to raise a weapon's *quality* if the services believe the original contractor is failing to fulfill performance requirements. According to Robert Drewes, this may have been one of the factors leading the Air Force to invite GE to compete against Pratt & Whitney for production of the F100 jet engine, "One of the most disturbing aspects concerned the Air Force perception that Pratt was more interested in generating profits through contract changes than in making the engine perform properly. . . . The Air Force leadership felt that if they went to Pratt to discuss technical problems, instead of meeting engineers and other F-100 program people, they would meet Pratt attorneys."[65] In testimony to Congress concerning the F-100 engine competition General Allen, Air Force Chief of Staff, responded, "We are anxious that Pratt and Whitney sees this situation as one in which their interests are best served by improving the durability and maintainability of the engine at acceptable costs. . . . The only issue at hand is what happens if they fail or are insufficiently motivated."[66]

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[64] Janos and Rich, 1994, pg. 331.
[65] Drewes, 1987, pg. 55.
[66] Drewes, 1987, pg. 101. House Appropriations Committee hearings on 15 February 1979, p. 463.

Production competition also provides alternative manufacturing sources for ensuring the uninterrupted supply of critical weapons components. Again remarking on the F-100 engine competition, Air Force General Robert D. Russ noted, "One of the things that drove us into this competition was the fact that we did not want to have almost our entire first-line fighter fleet with the same engine. You can imagine what would happen if we had catastrophic failure of the hot section in the engine. The entire force would be grounded."[67] Hence there is clearly a security argument, entirely independent of procurement costs, which can be made for dual-sourcing some critical weapon system components.

Regardless of the reason for bringing a new firm into production competition, it appears that once a second-source is introduced, they are rarely allowed to exit during subsequent competitions. In theory, the original developer should have both an experience advantage over the new rival and superior cost information, even after the technology transfer. Anton and Yao[68] champion the idea of second-sourcing as a signal by the government to prove it is committed to production competition as a means of forcing the initial developer to lower its price. However, Anton and Yao note that because the original developer has both an experience advantage and an information advantage, the new entrant should never

[67] *Department of Defense Appropriations for 1986*, Hearings, pt. 6, p. 295. Rpt in Pilling, 1989, pg. 29.
[68] Anton and Yao, 1987, 57-75.

win the reprocurement contract competition. But contrary to their theory, in the reprocurement programs examined by Anton and Yao, the new entrant won sixteen of the seventeen winner-take-all competitions.[69] One plausible explanation for this phenomenon is that it simply reflects a DOD bias for initiating reprocurement competitions when it is dissatisfied with the performance of the original developer. Jim Leitzel more disparagingly suggests that the DOD uses weapon reprocurements to spread contracts around to meet cartel distribution requirements because the DOD has been "captured" by the defense industry and merely operates to raise industry profits.[70] However, the DOD does have a vital interest in maintaining multiple production sources for some systems and a capable defense industrial base to respond to crises and facilitate future competition. Another convincing, but less dramatic explanation for why new entrants win contracts so often is offered by Ben Rich, former head of Lockheed's Skunk Works, when he argues that new rivals win because they do not have as high of research and development costs as those born by the weapon's initial developer.[71] For example, the R&D and tooling costs for the B-2 bomber totaled $32 billion which was included in the price of the first 20 bombers sold to the Air Force by Northrop.[72] However, Northrop is now offering to sell the Air Force an additional 20 bombers at a substantially reduced, guaranteed

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[69] Anton and Yao, 1990, pg. 65.
[70] Leitzel, pg. 49.
[71] Janos and Rich, 1994, pg. 331. Lockheed's Skunk Works produced such notable successes for the Air Force as the SR-71, the U-2, and the F-117 Stealth Fighter.
[72] "A Military Aerospace Survey," *1994, pg 7.*

fixed price of $570 million each having recouped its initial investment.[73] Thus while the developer seeks to recover its research and development investments, the rival is able to compete without the same burden because the technology was passed on to the rival by the government.

I am also inclined to believe that the government would find it politically very difficult to convince the press and the public that it made the right choice for second-sourcing if it paid millions of dollars to transfer the winning technology to a new entrant, only to allow the entrant to fail during the reprocurement competition In the public's eye, the original transfer of technology would appear to be a huge waste of resources. Thus, by choosing to second-source the DOD may already have politically committed itself to letting the new entrant succeed, perhaps at the expense of the incumbent. This inclination towards ensuring the success of the new entrant may also be manifest in the large number of instances in which the government chooses to split production contracts -- even when splits cost the DOD more than awarding the entire production contract to a single firm. In the F-100 engine competition, the Air Force chose a split award, even though the split was

[73] Although it seems sort of like an oxymoron to say "reduced price of $570 million," even this figure is higher than it needs to be because all the bombers will be produced at inefficiently low rates of production (three per year). In fact, the government is considering cutting that production rate to only 1.5 aircraft per year which would raise the cost of each bomber by an additional 25 percent, without counting for inflation. Also, these prices demonstrate the problem of comparing sole-source prices with later competitive prices -- the initial 20 bombers were given a unit cost of $830 million while the second 20 are priced at $570 million. In early studies, this reduction would have been claimed as a benefit of competition if there was a second firm when clearly the reduction came here without adding another firm. Morocco, *1994, pg. 62.*

forecast to cost approximately $1 billion more than a sole-source award to either Pratt & Whitney or GE.[74] Similarly, Beltramo found that in a number of split-buy competitions the higher-bidding producer was awarded the higher share of production (e.g., the Rockeye bomb, Shillelagh missile, and the Shrike missile) suggesting that factors other than cost are considered when making awards.[75]

Unfortunately, if the original developers feel they actually are at some sort of disadvantage (and if one believes Mr. Rich's comments, then it appears that they do), this would imply that second-sourcing leads to inefficient amounts of investments because developers fear the new entrants will reap the gains from the incumbent's investments. This is the crux of why Laffont and Tirole (1988) are pessimistic about the virtues of second-sourcing, stating, "In the case of *transferable* investment, any cost savings from the investment become savings for the second source if the first source is replaced. Because the investment is not observable and so cannot be compensated directly, the incumbent has insufficient incentive to invest; . . ."[76] Their recommendation is for the DOD to favor the incumbent on reprocurement bids -- an action which has clearly not been practiced

[74] Pilling, pg. 28. However, one should note that part of the price difference was in the way both companies priced their warranties -- offering large savings on the warranties if the firm was awarded the entire contract. One could argue, though, that the reason both firms offered large savings on warranties only in the event that they received the entire contract was because with the absence of a competitor, there would be little to judge engine performance against, and therefore the warranties would be much less enforceable.
[75] Beltramo, "Dual Production Sources in the Procurement of Weapon Systems: A Policy Analysis." RAND study #P-6911-RGI, 1983. Rptd in Anton and Yao, 1990, pg. 75.
[76] Laffont and Tirole, 1988, pg. 518.

with any regularity in the past.[77] Alternately, they propose allowing developers to retain the property rights to their technologies and/or data so the incumbent will know that its investments will be rewarded.[78] Notably, the 1986 Packard Commission also recommended that developers not be required to give up their property rights to new technologies if the product was developed with the firm's private funds.[79]

William Rogerson claims that *any* activity that reduces profits will discourage investment in research and development by defense firms.[80] Suggesting there may be a trade off between encouraging innovation and encouraging productive efficiency, Rogerson found that under sole source production, on average from 3.3 to 4.7 cents of every dollar of production revenue for defense contractors was pure economic profit.[81] Major defense contractors received revenues of $99.3 billion on defense contracts in 1992. Therefore, if these firms earned economic profits of 3.3 to 4.7 percent they earned roughly $3.3 billion to $4.7 billion in profits on their production contracts. However, defense firms also

[77] One might point out that often the government may attempt to reimburse contractors on a "cost-plus" basis so that they are not afraid to make investments. However, this leads to an entire new set of problems which will be discussed later in the paper.
[78] Even though it was never second-sourced, Lockheed developed the F-117 substantially with their own funds, and retained the rights to the design and technology. While Lockheed lost $6 million on the first five production aircraft, the program was such a success that the company eventually made about $80 million through large increases in production efficiency on subsequent orders. Janos and Rich, 1994, pg. 24, 85.
[79] *A Quest for Excellence,* 1986, pg. xxvi.
[80] Rogerson, 1989 and 1994.
[81] Rogerson, 1989, p. 1304.

privately funded $3.5 billion in research and development and bid and proposal expenditures (these figures for 1989). Thus, most of the production profits of firms were simply used to cover privately funded expenses on research and development and bid proposals for government contracts.[82] This implies that if the government introduces competition in the production phase specifically to reduce production profits, one of the primary effects of such an action would be simply to reduce the amount of research and development which takes place prior to production while leaving the firm's total profits relatively unchanged. Reinforcing this conclusion, Riordan and Sappington contend, "an additional disadvantage of second sourcing is that it discourages R&D effort and lengthens the development lag. . . . Hence with sole-source development the need to motivate R&D effort strengthens the case for sole-sourcing as opposed to second-source production."[83]

Considering both the pros and the cons of production competition, I am inclined to believe that dual-sourcing production should generally be avoided unless required for reasons other than cost reduction. It is during the production phase that firms recoup their investments made during the development of new weapons. Competition during production takes away the "prize" incentive which firms compete for at the beginning of the "tournament." Without a prize, there can be no effective competition. If firms know they will be forced to "compete away" profits

[82] Rogerson, 1994, pg. 65-90.
[83] Riordan and Sappington, 1989, pg. 55.

during production they will be less inclined to compete in price and effort during development which may ultimately lower quality and raise the cost to the government over the lifetime of the procurement. If the government wants to use competition to procure new weapons, it should commit to vigorous competition during the weapon's development where production costs and performance are designed into the system, but refrain from competition during production itself unless the contractor fails to meet prespecified cost or performance objectives which are clearly spelled out at the time of the competition. DOD and Congress must work to establish reputations for not always acting opportunistically after contractor investments are sunk. On the other hand, if the DOD believes certain components should be dual-sourced for reasons besides cost, then the government should explicitly state its intentions to dual-source production early in the competition so firms do not feel misled at the end of development. The government must be forthcoming about its intentions to dual-source or it cannot hope to establish a reputation that will support competitive efforts when it does not dual-source.

3.7: CONTRACTING AND FUNDING PROCUREMENT PROGRAMS

3.7A: Annual Budget Reviews

The instability and short-sightedness of the weapons funding process probably has had as much to do with the inefficiencies of the acquisition system and

the failure of competition as any other single factor. The annual budget review process affects the quality, quantity and timing of weapon purchases as well as the level of investment in capital and the very health of the defense industry. In the words of the Packard Commission,[84]

> The national defense program depends upon steady, long-term vision if it is to meet our long-term security needs effectively. Congressional focus, however, is myopic and misdirected. Only the upcoming budget year gets real attention, and this attention is directed at the budget's microscopic pieces, its line items.

According to the Commission's report, during the review of the 1985 defense budget, Congress made individual changes to over 1,800 separate defense programs and directed the DOD to conduct 458 studies ranging from the feasibility of selling lamb products in commissaries to the status of retirement benefits for Philippine scouts.[85] Of the five problems most often identified by industry as significant barriers to weapons procurement, two are direct results of the annual budgetary process: *the instability of the department's requirements and budget* and *the government's right to terminate contracts at will.*[86] According to defense analyst

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[84] *National Security Planning and Budgeting*, 1986, pg. 15.

[85] *National Security Planning and Budgeting*, 1986, pg. 15.

[86] The five barriers are: Unique laws and regulations imposed on government contractors; *the instability of the department's requirements and budget*; the imposition of government-unique rules on commercial subcontractors; *the government's right to terminate contracts at will*; and industry's perception that an inadvertent mistake by the firm could lead to criminal or civil penalties and ruin the good name of the company in the commercial marketplace.

Thomas McNaugher, "Whatever the technical risks defense contractors face in designing something that has never been designed before, it is the political risk that they really worry about: Are we going to get money this year?"[87]

One obvious problem with annual funding of procurement programs is that it can lead to delays and inefficient production rates which raises the cost of procuring weapons. For example, the *Air Force Times* reported that the cost of procuring the Air Force's new Advanced Tactical Fighter, the F-22, will cost $5.3 billion more than originally planned because of delays in development and production brought about by three years of spending cuts by Congress.[88] A 1987 study by the Congressional Budget Office examined 40 weapons systems produced between 1983-1987 and found that half of the systems were being produced below "minimum economic rates," and it was estimated a 50 percent increase in the production rate of those systems would have decreased average costs by ten to fifteen percent.[89]

Beyond the direct effects on costs and schedules, annual budgetary reviews also eliminate the ability of the DOD to offer a credible prize to the winner of any procurement competition. One 1983 RAND Corporation study found that "No

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Briefing provided by Secretary Perry to Senate Armed Services Committee on Feb. 24, 1994. *Defense Issues,* vol. 9, no. 10, pg. 2.

[87] Thomas McNaugher, "What's Wrong with the Way We Buy Weapons?" *1988,* pg. 6.

[88] According to the report, both Congress and senior Pentagon officials have tried to work within declining defense budgets by stripping $860 million from the F-22 program from the FY 1993 to FY1996 budgets. However, officials believe the inflation increases due to the delay will more than offset the cuts. Watkins, *1995.*

[89] Congressional Budget Office, "Effects of Weapons Procurement Stretch-Outs on Costs and Schedules," 1987 (Rptd in Rogerson, 1993)

major Air Force program has been procured to the original plan since 1969."[90] From the firm's perspective, why should it invest millions of its own dollars in innovative designs or cost-savings techniques if it risks having the program canceled or significantly scaled back at any time. Therefore, the annual budget review reduces the incentive for firms to privately invest in research and development and as a result, the government must finance much more of the capital and equipment used by private firms in the production of weapons. Today, the government owns most of the plant space and equipment of the prime contractors in the munitions and strategic missile industries, one-third of the capital in the aircraft industry, and much of the repair facilities in the ship industry.[91]

By acting opportunistically with the annual defense budget, Congress distorts the incentives of both the military and defense firms in the early stages of the acquisition cycle. The annual budgetary review process creates incentives for defense firms and the military to favor optimistic bids and proposals by defense contractors. As long as weapons programs appear to be reasonably priced and test results seem to be up to par, Congress is less inclined to interfere in the budget process. This leads firms to be highly optimistic about the cost and performance of new weapons. By underbidding costs and overstating performance, firms hope to

[90] Dews, Edmund and John Birkler, "Reform in Defense Acquisition Policies: A Different View," *RAND Paper P-6927, Santa Monica, CA., 1983*, pg. 3. (Rptd in Rogerson, 1993)
[91] Gansler, 1989, pg. 164.

create enough budgetary momentum in a weapons program to keep it from being cut in later phases when inevitable cost overruns occur. Once the government has already invested billions in developing a project, it often feels politically compelled to see the program through to completion rather than eliminating thousands of jobs, and admitting to a multi-billion dollar mistake.

The military has few incentives to intervene in this practice because their interests lie on the same side as the contractor for getting new weapons through Congress. If one program is canceled by Congress, the projected money from the canceled program is not offered back to the military to use on other more successful procurement programs. From the military's perspective, either the program is funded or the money is taken away. If those were your incentives, would you push to cancel inefficient programs or try to push them through and improve them as much as you could? Thomas McNaugher articulated this point well when addressing the problems of our procurement process:

> They [the services] understand my point that it takes an operating force to win wars. But they also understand the political point that it's very difficult to get these sophisticated development projects through the political process without some cost growth and other things that raise flags on Capitol Hill. So they rush the thing through and then they fix it out on the field.[92]

[92] McNaugher, "What's Wrong With the Way We Buy Weapons" 1988, pg. 8.

So the instability of Congressional funding creates perverse incentives for the military to support programs that it would not otherwise support if it had more proprietary control over its funding. In two related economic analyses, Rogerson shows that by being the first mover in the procurement process, the military may distort its decisions to alter the future marginal benefits and marginal costs of a program in order to manipulate future decisions by Congress.[93] Rogerson suggests the solution to this problem is for Congress to commit to a funding level for an entire program at the outset and not revisit the decision unless truly major problems occur. In this way, military has a credible "prize" to offer firms but agents do not have an incentive to manipulate the budget.

3.7B: Contracts and Negotiations

Whatever problems the DOD faces due to the instability of funding, they are greatly compounded by the inclination for negotiating what effectively become cost-plus procurement contracts. Significant changes resulting in renegotiations of the original contract are common in weapons procurement programs. One study found that renegotiation resulted in an average increase of 31.3 percent in the negotiated target price over the life of a typical contract.[94] Rogerson points out that

[93] Rogerson, *1994* and Rogerson, 1993.
[94] Joao Manuel Pacheco de Figueiredo, "The Dynamic Defense Process: The Role of Cost Growth, Cost Share Ratios, and Cost Overruns in Fixed-Price Incentive Contracts," UC Berkeley, Graduate School of Business, 1988. (Rptd in Rogerson, 1994, pg. 67)

government contracting officers are carefully instructed *not* to obtain the lowest price during renegotiations, but rather to negotiate a *fair* price, based on estimated costs, when dealing with prime contractors.[95] However, if defense contractors rationally anticipate the opportunity to renegotiate procurement contracts after being declared the sole-source producer, this creates inadvertent incentives for firms to be excessively optimistic about weapon costs and performance. According to McAfee and McMillan:

> A fixed-price contract or an incentive contract will become, in effect, an ad hoc cost-plus contract if the government is unduly willing to change the terms of the contract after it has been awarded. If the bidding firm believes it will be able to persuade the government to raise the agreed-upon price during the course of the project, then it will deliberately underbid.[96]

This practice has become so common in the defense procurement process that Walter LaBerge, former Under Secretary of the Army, metaphorically described the art of writing bid proposals as a game of "Liar's Dice" where the winner is most often the player who offers the most optimistic assertion that has a chance of being believed by someone not knowing the facts in detail.[97] This bidding strategy was corroborated by Lockheed executive Ben Rich, writing:

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[95] Rogerson, *1994,* pg. 69.
[96] McAfee and McMillan, 1988, pg. 37.
[97] As Dr. LaBerge explains, "The DOD acquisition system is very much like the barroom game of 'liar's dice.' In that game, winning comes from concealing the true facts (e.g., the roll of one's dice) and by asserting not what *is* but rather what *might be*. To win, the player must put forward

> At the heart of the defense industry problem was a recognition that if we bid unrealistically low to get a project, the government would willingly make up the difference down the line by supplying additional funding to meet increasing production costs. And it would do so without penalties.[98]

Indeed, contractor bailouts have occurred regularly in the past. As examples, during the 1960s, Lockheed was rescued in its contract for the C-5, in the 1970s Grumman was allowed to renegotiate prices for the F-14, and in the 1980s McDonnell Douglas and Northrop were saved in the production of the F-18.[99]

However, cost overruns and bailouts are not the worst economic problems of cost-plus contracting. The real danger is that there is no longer any assurance of selecting the most efficient producer. McAfee and McMillan explain:

> [T]he fault of the cost-plus contract is that . . . .the lowest-cost firm will not necessarily submit the lowest bid. The bidding process fails to reveal relative expected costs. If more than one firm bids, then it is likely that the government will select the wrong firm. Notice that this argument is different from the standard objection to cost-plus contracts: namely that a cost-plus contract gives the firm no incentive to keep its costs low. . . . The fact that the cost-plus contract defeats the very purpose of competitive bidding is a more fundamental objection to its use.[100]

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the most optimistic assertion that has a chance of being believed by someone not knowing the facts in detail. One who straightforwardly and honestly describes his situation rarely wins against the veteran liar's dice player." LaBerge, 1982, pg. 56.

[98] Janos and Rich, 1994, pg. 329.

[99] Stubbing, 1993, pg. 152.

[100] McAfee and McMillan, 1988, pg. 33.

Until the government establishes a reputation for sticking to negotiated contracts, any attempts at using competition to induce efficiency cannot be wholly successful. The government must find a means of making contract negotiations more binding and contracts must provide credible incentives for firms to report the truth and put forth effort to reduce costs.

Of course, the military's incentives for allowing contract renegotiations are intimately connected with the budget process and the fact that the military has little proprietary control over its procurement funds. Since the military would rather have an expensive weapon system than no weapon system at all, it is more than willing to renegotiate contracts. On the other hand, if the DOD had the option of rechanneling funds into other more successful programs or even if money saved could be used in the same program, it would be more likely to enforce the original contract terms, and contractors would be less willing to "buy in" to contracts.

Finally, before we leave the subject of contracts and budgets, it is important to point out that many of the problems we have discussed are by-products of the uncertainties of the development process. Contractors probably do not intentionally lie about their cost or performance forecasts, but at the same time they cannot be expected to predict the uncertainties of the future. In all probability, contractors can only estimate a cost and performance "range" when they begin the development program, and current incentives encourage contractors to report the costs that fall

on the low side of the range while reporting performance from the high side. One of the benefits of holding prototype development competitions, however, is that it allows contractors to substantially narrow the forecast range of cost and performance. Therefore, well-run prototype competitions should help reduce cost overruns and contract renegotiations, and also aid the government in selecting the best-qualified firm to produce the weapon.

3.8: TECHNOLOGY AND THE SOARING COSTS OF WEAPONS

Without a doubt, the greatest cause of public contention in military procurement over the past two decades has been the escalating costs of America's new high-tech arsenal. Between 1970 and 1990, the United States invested heavily in new, high-tech military weaponry. Throughout this period, there was intense debate in both parties and across the defense establishment on the military usefulness of America's expensive new arms. Many prominent analysts in and out of government openly criticized the DOD for purchasing overly sophisticated "gold-plated" weapons that were too expensive to buy in useful quantities and too sophisticated to be reliable in battle.[101] Burnett begrudgingly points out that the

[101] For a synopsis of the debate see William Perry, "Defense Reform and the Quantity-Quality Quandary" and Pierre Sprey, "The Case for Better and Cheaper Weapons" in The Defense Reform Debate, ed. Clark, A., Chiarelli, P., McKitrick, J., and J. Reed. (1984).

highly regarded P-51 Mustang of World War II cost only $300,000 while in the mid-1970s the F-15A Eagle cost about 27.5 million.[102] When faced with such stark statistics, it is easy to make an emotional case that our new weapons are far too expensive, regardless of their performance.

Most of the "defense reformers" complaints were based on American experiences in Vietnam when we had apparently become infatuated with the benefits of technology to the detriment of military capability. For example, American weapons buyers were so enamored with the emerging air-to-air missile technology that they did not even build a gun into the F-4 -- a tactical mistake which was not well received by American fighter pilots. (The Pentagon corrected the error by having the F-4s carry a gun pod beneath the aircraft, at a substantial cost in fuel efficiency and aircraft performance.) Additionally, while the cheap AIM-9 heat-seeking air-to-air missile performed reasonably well in the skies of Southeast Asia, the longer-range and much more expensive AIM-7 radar-guided missile performed miserably.[103] Based on these previous experiences with "new" technologies, defense reformers claimed the military would be far better served by relying on inexpensive, "tried and true" technologies purchased in larger quantities rather than fewer high-tech weapons of questionable reliability. Eventually, the popular press also picked

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[102] Both figures are in constant 1984 dollars. Burnett, 1987, pg. 20.

[103] Pierre Sprey notes that the AIM-7 had a kill rate of .08 to .10, less than one-third that of the cheaper AIM-9D/G. Sprey, 1984, pg. 200.

up on the debate calling for simpler, cheaper weapons -- although the prescribed lists of good versus bad weapons evolved strangely over time.[104] Just three months prior to the Persian Gulf War, Scott Shuger reflected the skepticism of many Americans towards our new high-tech arsenal in an article critical of the F-117 stealth fighter, writing, "at this date there is no reason to doubt that in a *brochure, laboratory sense,* the airplane is an accurate bomber. But there is a world of difference between that notion and *operational* accuracy, which is the accuracy achieved by a system when it is subjected to the 'fog of war.'"[105] In 1991, however, the Persian Gulf War would test America's new high-tech weapons in a real combat environment.

During a prolonged conflict with neighboring Iran in the 1980s, Saddam Hussein poured over $80 billion into the expansion and modernization of Iraq's army, amassing the fourth largest military in the world including 900,000 battle-tested ground troops organized in 60 regular divisions and 8 Republican Guard divisions; 16,000 surface-to-air missiles; 7,000 antiaircraft guns; 5,700 tanks; 5,000 armored vehicles; 3,700 artillery pieces and over 750 fighter, bomber and armed trainer aircraft.[106] By January 15, 1991, the eve of the first allied air attack,

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[104] For example the July 10, 1989 *U.S. News and World Report* article "Best and Worst Weapons" lists the SSN-688 nuclear submarine as one of the "best weapons" for cost effectiveness, but just five years earlier Pierre Sprey had called it one of the worst weapons and advocated canceling it in favor of older diesel submarines. Sprey, pg. 198.
[105] Shuger, "1990, pg. 46.
[106] *Reaching Globally, . . . ,* 1991, pg. 4.

Baghdad's defenses were denser than any Eastern European target at the height of the Cold War, and seven times more dense than Hanoi before the Linebacker II bombing campaign in December 1972.[107] As America approached confrontation with Iraq, many predicted a prolonged engagement with large numbers of allied casualties.[108] However, their concern was unfounded and America's new weapons proved to be survivable, reliable, and overwhelmingly effective in combat.

In over 30,000 sorties during the war, the U.S. Air Force lost just 14 aircraft for an overall loss rate of .047 percent.[109] By way of comparison, during the short Linebacker II campaign of December 1972, U.S. Forces flew 1,945 fighter and bomber sorties against less fortified targets near Hanoi and Haiphong and lost a total of 26 U.S. aircraft, including ten B-52 bombers.[110] During Israel's successful six-day war of 1967, flying American F-4 Phantoms and A-4 Skyhawks the Israelis attacked 28 Arab bases including H-3 in Iraq and suffered a loss rate 40 times greater than the Gulf War.[111] (Losses on the Arab side using less sophisticated Soviet equipment were higher despite having a quantitative advantage.) During World War II, the 8th Air Force suffered losses as high as ten percent -- more than

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[107] *Reaching Globally,* . . . ,1991.

[108] For example, Joshua Epstein of the Brookings Institution developed a model which predicted 3,344 to 16,059 casualties including up to 4,000 deaths and *Newsweek* predicted 5,000 dead and 15,000 wounded in the first ten days. Muravchik, 1991, pg. 19.

[109] *Reaching Globally,* . . . , 1991.

[110] Pasternak, 1991.

[111] *Reaching Globally,* . . . ,1991, pg. 34.

200 times greater than the Gulf War[112] and on some raids, the losses were truly staggering such as the second raid against Schweinfurt when the Air Corps sent 291 B-17 bombers and lost 60 aircraft and almost 300 airmen.[113]

During the Persian Gulf War, the Air Force's combat mission capability rates averaged close to 90%, exceeding peacetime rates for reliability.[114] As for mission effectiveness, during World War II the Eighth Air Force needed 108 B-17 bombers dropping a total of 648 bombs (six 1,100 lb. bombs per plane) to ensure a 96% chance of disabling a single Nazi power plant.[115] In Vietnam, an interdiction mission to destroy one bridge required hundreds of combat sorties. During the Gulf War however, a single F-117 could perform either mission in one sortie with one "smart" bomb. The advantages of the new stealth technology during the war are apparent when one observes that although stealth fighters composed only two percent of total allied air assets in action and flew only one percent of the total coalition sorties, they suffered zero losses, accounted for 40 percent of all damaged strategic targets and compiled a 75 percent direct hit rate, including the destruction of thirty-nine of the forty-three bridges spanning the Tigris-Euphrates river.[116] Given its performance, the $45 million price tag on the F-117 was a real bargain compared to the potential costs of performing those missions with conventional aircraft.

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[112] *Reaching Globally, . . . ,*1991, pg. 34.
[113] Bazley, 1986, pg. 10.
[114] *Reaching Globally, . . . ,*1991, pg. 35.
[115] *Reaching Globally, . . . ,*1991, pg 31.
[116] Janos and Rich, 1994, pg. 104

It is a myth that the allied success in the Gulf War was simply due to the sheer magnitude of the bombing campaign. In reality, the U.S. Air Force dropped *less* tonnage per month in the Gulf War than it did in either WW II or Vietnam. (47,778 tons of bombs per month in WW II; 44,014 tons per month in Vietnam; and 40,416 tons per month in Iraq) Total Air Force expenditure was less than four percent of that dropped on Germany in W.W.II, and less than one percent of that dropped in Vietnam.[117] The real difference in the war was in the accuracy and therefore the effectiveness of the Gulf War attacks. Furthermore, the new weapons allowed the U.S. military to prosecute the war against Iraq while largely avoiding the extensive collateral damage that may have been caused by hundreds of "dumb" bombs that would have normally missed their targets in the past. Besides shortening the war and saving American lives and dollars, the use of precision-guided munitions and delivery systems also reduced the number of Iraqi civilian casualties.

Despite the overwhelming results of the war, some analysts and economists remain skeptical about the value and the cost of modern high-tech weaponry. A year after the war, Jim Leitzel still lamented, "Why would the DOD systematically purchase weapons that are too complex, too expensive, and generally low quality if the DOD represents the users of those systems?" subsequently explaining in a footnote that "the astounding success [in the Gulf War] came *relative* to the Iraqi

[117] *Reaching Globally*, . . . , 1991, pg. 29.

military."[118] However, I believe Leitzel's explanation misses the point entirely. As has already been mentioned, the Iraqi military was a well-equipped and formidable foe. The reason Saddam Hussein offered such remarkably poor resistance was precisely because he underestimated the tremendous capabilities of America's new high-tech arsenal. The very fact that Iraq's large military was decimated in the desert while believing it could engage America in a prolonged war of attrition is the highest testimony one can pay to the revolutionary effectiveness of America's new military. Leitzel has inadvertently argued in favor of high-tech weapons. In a weapon-by-weapon rating of the performance of our high-tech Gulf War arsenal, former deputy under secretary of defense Dov Zakheim ultimately concluded:[119]

> What cannot be denied is that the general American emphasis on a combination of high technology and highly capable, well-trained volunteer forces paid off, not merely in victory, but at a human and material cost that was far lower than military leaders ever dared to imagine. Thus, at a minimum, the notion that somehow quantity, at lower levels of capability, can replace quality systems should be put to rest.

The Pentagon does not have to convince Congress that its new weapons are effective. Instead, the bigger problem for DOD is demonstrating that a sufficiently advanced military threat exists to justify continued improvement in our capabilities.

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[118] Leitzel, 1992, pg. 48. (My italics)
[119] Zakheim, *1991, pg. 21.*

Is technology the panacea of military procurement? Not entirely. While our more advanced systems did perform splendidly in the Gulf, the allies also held a distinct numerical advantage in the air, and some of our "simple and cheap" aircraft (such as the A-10) also performed remarkably well with and without the benefit of precision-guided munitions. However, commanders gave those aircraft decidedly different missions from those which were performed by others such as the F-117. It could be argued that one of the real success stories of the war was the synergistic integration of both high and low-tech weapons into a potent plan of battle which effectively used each. Thus, the military would probably be acting inappropriately if it solely pursued "high-tech" for its own sake, but it would be equally errant if it rejected the advantages of technology in favor of large quantities of cheap equipment. What is clearly needed is a balance of technology, cost, and mission objectives.

The cheap A-10 is a truly magnificent tank-killer in the right environment, but you certainly wouldn't want to send a squadron of A-10s to do the mission of the F-117. The inexpensive F-16A armed with a couple of Sidewinders and a cannon truly is a remarkable dogfighter if it reaches the merge, but this simple combination cannot perform all the roles required of a modern air superiority fighter as suggested by many reformers.[120] The new AMRAAM is an expensive missile to

[120] In the Gulf War, the majority of the U.S.'s 31 air-to-air victories came from medium-range radar-guided missiles (Westlake, *1991, pg. 49)* and not short-range heatseekers.

employ in combat, but it costs far less than the aircraft and the pilot at the receiver's end of the weapon. Moreover, the AMRAAM allows our pilots to engage and destroy multiple enemy aircraft before our forces are attacked. This capability, not available on older, simpler systems like the Sidewinder, is an incredible force multiplier -- in effect saving the cost of purchasing additional aircraft and pilots for the cost of a missile. Therefore, the focus should not be on technology, nor should it be on individual weapon costs. Instead, we should scrutinize *the cost of performance*. That is the area where technology should reduce the price of military effectiveness, and it is also the area where the current acquisition system falls short.

As Secretary Perry has noted and emphasized, electronics is one area where the military can and should lower the cost of performance --while the price of a car has risen from $2,000 in 1960 to $20,000 today, the cost of a computer has dropped from a few million dollars to a few thousand for the equivalent capability.[121] Across the board, inflation and improvements in performance have driven up the price of virtually every product except electronics where breakthroughs in research and large consumer demand have reversed the trend toward higher prices. Electronics also offer the military incredible capabilities in targeting, communications, identification, and other tactical and strategic areas of concern -- many of which became evident in the Gulf War. Yet, electronics have not been getting cheaper in military systems and

[121] See Perry, "Defense Reform and the Quantity-Quality Quandary," 1984.

the decade-long acquisition cycle makes many new systems obsolete before they are ever deployed in the field.

A recent Defense Science Board study of comparable electronic systems found the commercial versions to be between two and ten times cheaper, up to five times faster to acquire, generally more reliable, and one to three years more advanced in technology than the military versions.[122] As an example, in the months leading up to Desert Storm the Army needed a large number of Global Positioning System (GPS) receivers for identifying their location on the desert battlefield. At the time, the service was told the "mil-spec" receiver would take 18 months to procure, weigh 17 pounds, and cost $34,000 apiece. However, high-quality GPS receivers that satisfied military requirements and weighed only three pounds apiece were bought on the commercial market in only a few months for $1,300 each.[123] Occasionally high military costs may be due to special military requirements such as nuclear hardening, but often, as in the case with these GPS receivers, such severe requirements are not necessary and the civilian component is more than adequate to meet the military's needs. Indeed, the military has a propensity for overspecifying even the most mundane items. Gansler reports that military specifications for fruitcakes sold in military commissaries take up 18 pages of minutiae, the

[122] Gansler, *1988, pg. 70.*
[123] News briefing by Defense Secretary William Perry, June 29, 1994. *Defense Issues*, vol. 9, no. 57, pg. 2.

specifications for towels take up 20 pages, 16 pages are devoted to plastic whistles, 17 pages to chewing gum, and the list continues almost endlessly.[124] Such detailed accounting and production requirements confound manufacturers and eliminate the opportunity to reduce costs through innovation and competition.

The DOD's recent push to streamline acquisition procedures and allow the increased purchase of commercial items should help reduce many of the costs of military-unique purchasing. In 1994, the DOD estimated that using commercial off-the-shelf technology would save up to $700 million over the next two to four years on microelectronics in the Army's new training helicopter alone.[125] Ultimately, if defense reform is to succeed, reforms must be broadly applied to these big-ticket procurements where DOD spends the vast majority of its procurement budget.

General George S. Patton, Jr., has written, "Never tell people how to do things. Tell them what to do and they will surprise you with their ingenuity."[126] The DOD would serve itself well to remember those words when it comes to buying new weapons. Once the military identifies a mission need and calls for weapons proposals, it should refrain from listing overly detailed performance *requirements*. By requiring the last few percent of performance improvement the cost of a new weapon system may go up by an additional 30 to 50 percent.[127] For example,

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[124] Gansler, 1989, pg. 191-192.
[125] *Defense Issues,* vol. 9, no. 57, pg. 1.
[126] Patton, 1947, pg. 357.
[127] Gansler, 1988, pg. 70.

Kenneth Mayer reports that the Air Force added hundreds of millions of dollars to the cost of the F-15 program by insisting that it be able to fly at Mach 2.5 instead of Mach 2.3 which could be attained relatively easily. Yet the additional 150 miles per hour at the top end is relatively meaningless to the fighter's overall tactical performance.[128] The DOD should clearly specify its performance preferences, including cost as a major design criteria, then see what contractors can do.[129] Allow contractors the freedom to create innovative designs and build prototypes that best fit their manufacturing capabilities by exploiting their most cost-effective technologies. This does not mean the DOD should be cut out of the weapons design process. On the contrary, the services should work closely with individual contractors so both can observe the cost and performance trade-offs from the manufacturer's and the user's perspectives. By clearly specifying preferences firms will know which innovations to pursue in order to have the best opportunity to win the prize, but firms can still make the judgment calls necessary to exploit their best capabilities. In contrast, by formalizing objectives as inflexible, mandatory requirements we greatly reduce the options firms have for competing against one another. The government must trust firms to take the innovative risks that lead to reduced costs *and* improved performance. The outstanding success of the F-117

[128] Mayer, 1991, pg. 47.

[129] Note that the military's "Design-to-Cost" program initially instituted in 1975 establishes cost as a parameter equal in importance with technical requirements and schedules. For a brief history of reform efforts in cost goals and estimates see Acker, *1982. pg. 75.*

stealth did not grow out of detailed Air Force requirements and oversight. On the contrary, Lockheed's design was the brainchild of a thirty-six year old Skunk Works mathematician named Denys Overholser who got his insight from a technical paper on radar published in Moscow nine years earlier.[130] Such innovative creativity would have been stifled by inflexible design requirements. Design specifications should be regarded as flexible preferences until the prototype competition is complete. However, once a prototype winner is chosen, design changes should be minimized to reduce the number and incidence of expensive contract renegotiations. Once again, this is an argument for the importance of building and testing prototypes to ascertain the risks and reliability of new technologies and creative designs. However, now we are emphasizing the need to give firms the freedom during prototype testing to use innovative ideas rather than requiring rigid, military-unique specifications. Ingenuity simply cannot be legislated.

3.9: CONVERSION

The buzzword of the 1990s is "conversion." As the defense budget plummeted in the early 1990s, it became clear that America could no longer afford to maintain the same defense industry that it had supported during the Cold War.

[130] Janos and Rich, 1994, pg. 19-24.

By 1992 employment in defense-related industries had already fallen by about 15 percent,[131] and by the turn of the century defense analysts expect that 80 of the top 100 defense firms will have quit the defense business.[132]

The fall in employment and the rapid consolidation of the defense industry has provoked concerns that industry contraction would gravely injure the nation's ability to respond to national crises and possibly cause long-term damage to the overall U.S. economy. Therefore, instead of simply allowing defense firms to fail and exit the business, the government has scrambled to help companies adapt to the commercial marketplace in hopes of saving jobs and preserving an adequate defense-capable industrial base. To this end, the Clinton administration has committed almost $2 billion over four years to support the conversion of defense R&D to the manufacturing of commercial products.[133] This consolidation and conversion of the defense industry seems to pose unique problems and opportunities for the DOD and the government.

The magnitude and pace of change in the defense industry can be illustrated by the recent flurry of mergers and acquisitions in the defense aerospace industry. In 1993, Lockheed acquired General Dynamics' aircraft division, while Martin-Marietta purchased General Electric Aerospace. In 1994 Northrop acquired

[131] *Defense Issues, vol. 9, no. 65, pg. 1.*
[132] "Military Aerospace Survey," 1994, pg. 4.
[133] "Critical Issues in Defense Conversion," Panel Report from the Center for Strategic & International Studies, 1994, pg. 23

Grumman, and Martin-Marietta purchased General Dynamics space launch division.[134] Already in 1995, the FTC has approved the $23 billion merger of Lockheed and Martin-Marietta.[135] Today, there are five U.S. companies that have demonstrated the ability to develop and produce modern military aircraft: Boeing, Lockheed-Martin, McDonnell-Douglas, Northrop-Grumman, and Rockwell.[136] By the end of the decade we may have only two or three.

The primary acquisition concern of many arising from the consolidation of the defense industry is a reduction in competition. However, in theory fewer firms should not have a direct impact on the ability of the DOD to induce competition because in most major programs the government ideally should not have more than two, or possibly three, firms competing for the prize. However, the consolidation causes an indirect problem because the failure of any single firm now has a larger impact on the entire industry. Therefore, with fewer firms the DOD may feel a greater compulsion to equitably distribute contracts among the survivors to prevent further exits. This problem, however, is not new. Casual empiricism suggests the DOD has always been careful to keep its prime contractors solvent. Stubbing and Mendel point out that of the top ten defense contractors in 1963, eight were still in

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[134] Kovacic and Smallwood, 1994, pg. 92.

[135] "Lockheed-Martin Deal Wins Approval," *Aviation Week & Space Technology*, January 16, 1995, pg. 25. For a more thorough discussion of the Lockheed-Martin merger and its projected impact, see Velocci, Sept. 5, 1994, pg. 36-42

[136] For a good discussion on competition and defense mergers in defense and specifically the aerospace industry, see Kovacic and Smallwood, 1994, pg. 91-110.

the top ten in 1988 and the other two remain in the top fifteen.[137] Ben Rich of Lockheed similarly acknowledges, "The open secret in our business was that the government practiced a very obvious form of paternalistic socialism to make certain that its principal weapons suppliers stayed solvent and maintained a skilled workforce."[138] If the government has been preoccupied with keeping eight or ten firms solvent in previous decades, then with fewer firms the problem should be about the same magnitude today, even though we have a smaller budget. Unfortunately, the practice of "paternalistic socialism" or even the *belief* that it is practiced by the government severely damages competition. Why should Lockheed invest its money and effort trying to win the contract for the B-2 if it is Northrop's turn to win? Why should Northrop invest time and money in developing the aircraft if they know that Lockheed is not investing because Lockheed knows its Northrop's time to win?

Oddly, however, the consolidation and conversion of the defense industry substantially alleviates the distribution problem. Economically, the *best* time to conduct procurement competitions is when the industry is expected to contract. A contracting industry creates a "buyer's market" and since the government is committed to letting some firms exit the industry, there is no thought that it will act

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[137] Mendel and Stubbing, *1989, pg. 55.*
[138] Janos and Rich, 1994, pg. 306.

to save every company in need.[139] However, the government should carefully supervise the consolidation of the defense industry to ensure that critical technologies are preserved in a manner that allows for continued rivalry between firms.[140]

In a news briefing, Secretary of Defense William Perry suggested, "integrating the defense industrial base into the national industrial base is an idea whose time has come."[141] Fortunately, the successful conversion of defense firms to the commercial marketplace will also help the government be a more credible sponsor of procurement competitions. Mendel and Stubbing inadvertently make this point in their criticism of DOD practices, writing:[142]

> Of the top aerospace companies today [1989], only one -- Boeing -- has been consistently successful in commercial markets. Boeing has a poor track record, however, in bidding on major new defense programs -- it has lost out as a prime contractor in six aircraft and missile programs in the past twenty-five years. In every case the Boeing proposal was highly competitive on technical and cost grounds, and Boeing's performance record on similar programs was satisfactory. But the winning contractors were in poor financial health or faced a

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[139] Conversely, when the defense budget is growing and the industry is booming as it was during the 1980s, it is much more difficult to establish effective competition because each firm knows that a better deal will eventually come its way if it is patient.

[140] For a fine exposition of maintaining industry rivalries and competition through during downsizing see Kovacic and Smallwood, *1988, 91-110.*

[141] *Defense Issues,* Vol. 9, No. 57, pg. 1.

[142] Mendel and Stubbing, *1989, pg. 55,56.*

declining business base, whereas Boeing was in excellent shape financially and had a large commercial base.

If defense firms can successfully penetrate the commercial marketplace, the DOD will feel less compelled to support them with equitably distributed government contracts. Therefore, successful conversion will affect the beliefs of the industry about the amount of competition supported by the government. This kind of change in industry beliefs should increase the overall level of competitive effort by rival firms. So, while many have expressed concern over recent industry trends toward consolidation and conversion, my analysis suggests these programs should not seriously hurt procurement competition, and may actually contribute to more effective competitions in the future.

3.10: CONCLUSION

Competition can be an effective basis for conducting procurement programs. However, past DOD efforts at inserting competition into the maze of existing acquisition policies have not been wholly successful because of major departures from the requirements of efficient research tournaments. Annual budget reviews by Congress and the threat of reducing firm profits by introducing new competition during production have handicapped the DOD's ability to offer a fitting prize to the winning contestant. The government's undue willingness to renegotiate fair profit

contracts has reduced the incentives of firms to invest in cost-saving innovations and more importantly has created the "buy in" implying that the government can never be sure it is selecting the most efficient contractor. Excessively detailed requirements and intensive oversight of contractor activities have prolonged the acquisition cycle, prevented the use of available off-the-shelf commercial substitutes, and diminished the ability of contractors to search for cost-effective innovations. Concluding competition prior to the building and testing of satisfactory prototypes has limited the amount of information available to all participants and eroded the decision-making process because too much uncertainty still remains when choosing the winning firm.

The theory behind efficient competition in research tournaments is really quite simple. Contrary to the beliefs of many analysts, effective competition does not require a great number of firms to drive down prices. The most effective competition for large projects will require only a few firms, but also a strong commitment by the government to reward the winning firm with a valuable prize. If government is committed to competition as a means of procuring new weapons, it should legally bind itself to appropriately awarding the winning firm so long as major cost and performance milestones are met. Congress needs to lengthen its review process and fund new weapons for multi-year periods up front, preventing

themselves from opportunistically exploiting contractor investments unless significant problems become evident during the procurement.

The DOD should commit to full-scale development prototype competitions as a means of increasing the available information so that it can select the most qualified firms, write more enforceable contracts, and demonstrate to Congress that the country is purchasing a cost-effective weapons system which will allow Congress to feel more secure in its longer term budget commitments. The DOD should eliminate inflexible detailed requirements in the early stages of the procurement process and rely on more personal exchanges of information with contractors so the user's and the builders can make better decisions on the costs and the capabilities of new systems.

We've previously gotten this process backwards. In the past we've written detailed and inflexible requirements in the design stages only to relax them and be quick to renegotiate contracts during production. Optimally, we should allow more flexibility for innovation during design and prototype testing, and be less willing to reward errors during production.

Oversight and detailed requests for accounting, cost, and technical information should not be requested from contractors unless absolutely necessary, because they only detract from the competition. Under the current process, the burden of oversight is placed on contractors independent of their performance.

Instead, oversight should only be used as a form of "punishment" if contractors fail to perform. The government should not be overly concerned about contractor profits, rather it should be concerned about whether or not the taxpayer's are getting a good value for their money.

Though Secretary Perry has called for radical changes in the acquisition process, past experience suggests reform will almost certainly be gradual, requiring the DOD to start with modest programs first. When dollar sums are relatively small, the government may be more willing to commit to competition without fear of seriously injuring the defense industry or national security. Once competition has been proven to be effective at reducing costs and improving performance on smaller procurements, there is a greater likelihood that competitive procurements will be politically feasible for larger programs. The Federal Acquisition Streamlining Act of 1994 is representative of the kind of reform that is needed and both economically and politically feasible today. By raising the threshold for simplified acquisition procedures from $25,000 to $100,000, decreasing the reliance on "mil-specs," and relaxing accounting requirements when suitable commercial products are available, the act reduced some of the expensive red tape and defense-unique overhead costs in military procurement. [143] Ultimately though, reforms must be extended beyond these "small" purchases to include major weapons systems because the vast majority

[143] Federal Acquisition Streamlining Act of 1994, signed October 13, 1994, (Public Law No. 103-355). *Weekly Compilation of Presidential Documents*, Oct. 17, 1994, v30, n41, p. 2000 - 2004.

of DOD acquisition dollars are spent on big-ticket items. Debating the Federal Acquisition Streamlining Act, the House Armed Services Committee noted that while the streamlining act will exempt an additional 55 percent of Pentagon contracts from the more stringent regulations applied to larger contracts, in terms of the procurement budget the act affects only about 5 percent of Pentagon contracting dollars.[144] Clearly, real progress cannot be made at reducing the waste in the procurement budget until necessary reforms are also applied to major purchases.

The basic requirements for running an effective research tournament are not that difficult to understand or implement. The biggest challenge acquisition reform advocates will face in their task is getting a real commitment to these simple requirements. To effectively use competition in the procurement process, the government must be able to credibly commit to awarding a prize to whichever firm wins the contest. The government must be able to limit the number of competitors to induce greater effort by each contestant. DOD must clearly specify its preferences so contractors know which innovations are most likely to be rewarded at the end of the tournament, but not be so restrictive as to prevent firms from using innovative ideas and designs to improve the competition. And finally, in the end the government only has to pick the innovation/bid combination that it believes is the most cost-effective purchase.

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[144] Towell, 1994, pg. 901.

CONCLUSION

It is difficult to accurately model the distribution and flow of information in economic theory. Often even the simplest economic problems can only be modeled under full and complete information, or by making extreme assumptions about the nature of the distribution of information. In the tournament literature, this constraint leads to a theoretical understanding of what an optimal contest should look like in a perfect world, but a recognition that insufficient information may lead to large deviations from the ideal scenario. In real life, tournament sponsors simply cannot be expected to know all that they need to know in order to conduct an efficient tournament.

At first glance, auctions are appealing mechanisms if only because they do not require vast amounts of information to implement. The primary purpose of this dissertation was to see if auctions could be usefully incorporated into tournaments to ease the information burden on sponsors and make it easier to conduct reasonably efficient tournaments. In chapter one I showed that auctions can be used to reduce or eliminate the problem of adverse selection of tournament contestants so sponsors can restrict entry to only the best qualified contestants. In chapter two I demonstrated how auctions can be incorporated at the end of the tournament to raise the equilibrium effort level of contestants and reduce the expected cost of the

prize. In both cases, auctions appear to be viable mechanisms for improving tournaments while simultaneously lowering the burden of information on the sponsor. So, in my opinion, auctions deserve serious consideration for improving the feasibility and efficiency of tournaments in practical applications, and merit further theoretical analysis.

The one practical application examined in this dissertation was defense acquisition in chapter three. Defense procurement and the defense industry in general are very complex economic environments, so I am hesitant about drawing too strong of conclusions based on the simple tournament models I have analyzed so far. Nevertheless, I do think auctions can be beneficially incorporated into the competitive acquisition process of defense procurement, but first I believe the government must take steps to make its acquisition policies more consistent with a competitive tournament environment. Current procurement practices appear to deviate substantially from the most basic requirements of any efficient competition. I am not bold enough to suggest that auctions and research tournaments are a panacea to all of the government's procurement problems or to the problems of private industry. On the contrary, even with auctions the uncertainties and risks in many procurement programs may be so large as to warrant a regulated approach to development rather than a competitive approach. What I hope this paper has done, however, is help clarify the important issues and problems that remain in

tournaments, and demonstrate a method which could be potentially useful for solving some of them so government officials and industry executives can be better informed when making decisions about using competition to solve their economic problems.

Appendix 1A

Lemmas and Theorems of Chapter 1

For the following lemmas and theorems, chapter one's notation is extended as follows:

x_i = the innovation of firm i.

$x_{m:n}$ = the mth best of n innovations.

$B(x_i)$ = any equilibrium bid of contestant i with x_i

To shorten notation, we may sometimes write $B(x) = b$ or $B(x^*) = b^*$.

We denote the Mth largest bid to be $b_{m:n}$

By assumption, the sponsor does not accept any bids less than zero because any firm with positive x can always bid zero and submit his endowment if selected.

$\theta(b_i)$ = the unconditional probability of gaining entry with a bid of b_i.

$\Gamma(b)$ = expected payment upon entry: $\forall\, b^*, \tilde{b} \quad b^* > \tilde{b} \Rightarrow \Gamma(b^*) \geq \Gamma(\tilde{b})$

$$\forall\, b^*,\ \varepsilon \geq 0,\ \Gamma(b^*) \leq b^* \text{ and } \Gamma(b^*+\varepsilon) \leq \Gamma(b^*) + \varepsilon$$

Therefore, $\Gamma(b)$ can represent either a uniform-price or discriminatory-price auction

$\Lambda(x_i, b_i)$ = the joint probability of gaining entry in the tournament with a bid of b_i and then winning the tournament with x_i given that you gained entry with the bid b_i.

$\lambda(x, b)$ is the associated pdf of Λ.

$\beta(b^*)$ = the cumulative distribution of the Mth highest bid given an equilibrium bidding function. So, $\beta(b^*)$ = the probability that the Mth highest bid is less than or equal to b^*.

$G(b^* | b_{m:n})$ = the conditional probability of gaining entry with a bid of b^* given that the Mth highest bid is less than or equal to $b_{m:n}$. If the distribution of equilibrium bids is continuous with no mass points $G(b^* \mid b^*) = 1 \ \forall\, b^*$ because

there is zero probability of any tie bids at b^*, implying b^* is large enough to be one of the highest M bids.

Therefore, $\theta(b^*) = G(b^* \mid b^*) \times \beta(b^*)$. Which says the unconditional probability of gaining entry with a bid of b^* equals the probability of gaining entry with a bid of b^* given that the Mth highest bid is less than or equal to b^* multiplied by the probability the Mth highest bid is less than or equal to b^*. If the distribution of equilibrium bids has no mass points then this reduces to $\theta(b^*) = \beta(b^*)$ so that the probability of gaining entry with a bid of b* equals the probability that the *Mth* highest bid is less than or equal to b^*.

$W(x \mid b^*)$ = the conditional probability of a firm winning the tournament with x given the firm gained entry with bid b^*. So, $\Lambda(x,b^*) = G(b^* \mid b^*) \times \beta(b^*) \times W(x \mid b^*)$

$\Omega(b \mid b^*)$ = the cumulative distribution of equilibrium bids whose population is *truncated* on the right at $B(x^*)=b^*$, so $\Omega(\hat{b} \mid b^*)$ is the probability that the sponsor receives a bid less than or equal to $\hat{b}$, *given* that all bids are less than or equal to b^*.

From these definitions, a firm's *ex ante* expected profit is:

$$E\pi(x,b) = P\Lambda(x,b) - \Gamma(b)\theta(b)$$

The following lemma is used repeatedly in the paper. This lemma was first proven in the generality shown here by Guesnerie and Laffont (1984); however, special cases were used by several authors, notably Myerson (1981) prior to this. Subscripts denote partial derivatives.

Lemma A1: *Suppose v: $[a,b] \to \Re$ is twice continuously differentiable. Then*

$$(\forall r)(\forall x)\ \ v(r,x) \le v(x,x) \text{ implies} \qquad \text{(A1)}$$

$$(\forall x)\ \ v_1(x,x) = 0 \text{ and} \qquad \text{(A2)}$$

$$(\forall x)\ \ v_{12}(x,x) \ge 0. \text{ Moreover, (A2) and} \qquad \text{(A3)}$$

$$(\forall r)(\forall x)\ \ v_{12}(r,x) \ge 0 \text{ imply (A1).} \qquad \text{(A4)}$$

First, I will use the theorem by Guesnerie and Laffont to show that there is no differentiable pure-strategy equilibrium for the tournament with T = 0.

The probability a single entrant wins the contest, given entry, is:

$$\Pr(\text{Win}|\text{Entry}) = \left(\frac{F(x_i) - F(y)}{1 - F(y)}\right)^{M-1}$$

If there are (N+1) firms competing for M entries, in order to gain entry a firm must bid higher than the Mth highest of the other N competing firms. The density of the Mth highest innovation of N firms is just the pdf of the Mth order statistic out of N draws:

$$h_{M:N}(x) = \frac{N!}{(N-M)!(M-1)!}\left[F(x)\right]^{N-M}\left[1 - F(x)\right]^{M-1} f(x)$$

With a strictly increasing function, $B(x)$, this is the density of the "critical" value, y. If a firm of type x_i follows the equilibrium bidding strategy it earns expected profits:

$$\pi\left(b(x_i), x_i\right) = P \int_0^{B^{-1}(b)} \left(\frac{F(x_i) - F(y)}{1 - F(y)}\right)^{M-1} h_{m:n}(y)dy - \int_0^{B^{-1}(b)} B(y)h_{m:n}(y)dy$$

Interpreting the previous expression, it is the prize (P) multiplied by the probability of entering and winning if firm i bids its equilibrium bid of $b = B(x_i)$, minus the expected entry cost. Thus, it corresponds directly to our expected profits:

$$\pi(b,x) = P\Lambda(x,b) - \Gamma(b)\theta(b)$$

Suppose a firm of type x_i chooses to deviate from the equilibrium strategy and bid as type $r < x_i$, then its expected profits are:

$$\text{(1)} \qquad \pi(r, x_i) = P\int_0^r \left(\frac{F(x_i) - F(y)}{1 - F(y)}\right)^{M-1} h_{m:n}(y)dy - \int_0^r B(y)h_{m:n}(y)dy$$

If a firm of type x_i chooses to deviate from the equilibrium strategy and bid as type $r > x_i$ its expected profits are:

$$(2) \qquad \pi(r, x_i) = P\int_0^{x_i} \left(\frac{F(x_i) - F(y)}{1 - F(y)}\right)^{M-1} h_{m:n}(y)dy - \int_0^r B(y)h_{m:n}(y)dy$$

Note that (2) differs from (1) only in the upper limit of integration in the first term. This is because when $r > x_i$, even though the firm gains entry at some values of y > x_i, it cannot win since y > x_i implies that all other entrants in equilibrium have larger values of x than firm *I*.

If *B(x)* is an equilibrium bidding function, then $B^{-1}(b) = x_i$ must maximize firm *i*'s expected profits. The first order conditions for a maximum require $\pi_1(x_i, x_i) = 0$. Checking the first order conditions for (1) and (2) gives us:

$$(1a) \qquad \pi_1(r, x_i) = P\left(\frac{F(x_i) - F(r)}{1 - F(r)}\right)^{M-1} h_{m:n}(r) - B(r)h_{m:n}(r)$$

$$(2a) \qquad \pi_1(r, x_i) = -B(r)h_{m:n}(r)$$

In both cases, evaluating at r = x_i gives us:

$$\pi_1(x_i, x_i) = -B(x_i)h_{m:n}(x_i)$$

Setting $\pi_1(x_i, x_i) = 0$ requires B(x_i) = 0. But it cannot possibly be an equilibrium for all firms to bid zero because then any firm with a positive x could do better by bidding a positive epsilon amount and ensuring entry for itself.

Lemma 1.1: *In a symmetric equilibrium, the distribution of bids contains no mass points.*

Proof: Assume ∃ a symmetric equilibrium with at least one mass point.

Let b^* be the uppermost mass point such that $\exists\, X = \{x \in [0,\bar{x}] \mid \Pr[B(x) = b^*] > 0\}$ and $\Pr[x_j \in X] > 0$.

Consider the equilibrium bid of a firm holding some arbitrary x^* interior to X.

By assumption, $\Pr[B(x^*) = b^*] > 0$.

Equilibrium requires:

$$P\Lambda(x^*,b^*) - \Gamma(b^*)\theta(b^*) \geq P\Lambda(x^*,\tilde{b}) - \Gamma(\tilde{b})\theta(\tilde{b}) \quad \forall\ \tilde{b} \neq b^*$$

A mass point at $b^* \Rightarrow G(b^* \mid b^*) < 1$ and $\{1 - G(b^* \mid b^*)\} > 0$.

By definition: $\Lambda(x^*,b^*) = G(b^* \mid b^*) \times \beta(b^*) \times W(x^* \mid b^*)$

Consider an alternate bid of $b^* + \varepsilon$, for some $\varepsilon > 0$.

Since b^* was the uppermost mass point, there is no mass point at $b^* + \varepsilon$.

$b^* + \varepsilon$ is not a mass point $\Rightarrow \Lambda(x^*,b^*+\varepsilon) = \beta(b^*+\varepsilon) \times W(x^* \mid b^*+\varepsilon)$

By definition: $\beta(b^*+\varepsilon) \times W(x^* \mid b^*+\varepsilon) \geq \beta(b^*) \times W(x^* \mid b^*)$

By definition: $\beta(b^*) \times W(x^* \mid b^*) = [G(b^* \mid b^*) + \{1 - G(b^* \mid b^*)\}] \times \beta(b^*) \times W(x^* \mid b^*)$

Substituting: $\beta(b^*) \times W(x^* \mid b^*) = \Lambda(x^*,b^*) + \{1 - G(b^* \mid b^*)\} \times \beta(b^*) \times W(x^* \mid b^*)$

Substituting: $\Lambda(x^*,b^*+\varepsilon) \geq \Lambda(x^*,b^*) + \{1 - G(b^* \mid b^*)\} \times \beta(b^*) \times W(x^* \mid b^*)$

By definition: $\Gamma(b^*+\varepsilon) \leq \Gamma(b^*) + \varepsilon$

$$P\Lambda(x^*,b^*+\varepsilon) - \Gamma(b^*+\varepsilon)\theta(b^*+\varepsilon) \geq P\Lambda(x^*,b^*+\varepsilon) - \Gamma(b^*)\theta(b^*+\varepsilon) - \varepsilon\theta(b^*+\varepsilon)$$

Substituting in for $\Lambda(x^*,b^*+\varepsilon)$ on the right-hand side we get:

$$P\Lambda(x^*,b^*+\varepsilon) - \Gamma(b^*)\theta(b^*+\varepsilon) - \varepsilon\theta(b^*+\varepsilon) \geq$$

$$P\Lambda(x^*,b^*) + P\big[\{1 - G(b^* \mid b^*)\} \times \beta(b^*) \times W(x^* \mid b^*)\big] - \Gamma(b^*)\theta(b^*+\varepsilon) - \varepsilon\theta(b^*+\varepsilon)$$

Therefore, equilibrium requires:

$$P\Lambda(x^*,b^*) - \Gamma(b^*)\theta(b^*) \geq$$

$$P\Lambda(x^*,b^*) + P\big[\{1 - G(b^* \mid b^*)\} \times \beta(b^*) \times W(x^* \mid b^*)\big] - \Gamma(b^*)\theta(b^*+\varepsilon) - \varepsilon\theta(b^*+\varepsilon)$$

Cancelling common terms and rearranging, equilibrium requires:

$$\Gamma(b^*)\theta(b^*+\varepsilon)+\varepsilon\theta(b^*+\varepsilon)-\Gamma(b^*)\theta(b^*) \geq P\big[\{1-G(b^*|\, b^*)\}\times \beta(b^*) \times W(x^*|\, b^*)\big]$$

Let $\theta(b^*+\varepsilon)=\theta(b^*)+\delta\theta$, then rearrange to get the equilibrium requirement:

$$\varepsilon \geq \frac{\left\{P\big[\{1-G(b^*|\, b^*)\}\times \beta(b^*) \times W(x^*|\, b^*)\big]-\Gamma(b^*)\delta\theta\right\}}{\theta(b^*+\varepsilon)}$$

As $\varepsilon \rightarrow 0$, the right hand side converges to

$$\frac{\left\{P\big[\{1-G(b^*|\, b^*)\}\times \beta(b^*) \times W(x^*|\, b^*)\big]\right\}}{\theta(b^*)}$$

which is strictly greater than zero.

Therefore, $\exists\ \varepsilon > 0$ such that

$$\varepsilon < \frac{\left\{P\big[\{1-G(b^*|\, b^*)\}\times \beta(b^*) \times W(x^*|\, b^*)\big]-\Gamma(b^*)\delta\theta\right\}}{\theta(b^*+\varepsilon)}$$

which contradicts our requirements for an equilibrium bidding function.

Therefore, there cannot be a mass point in any symmetric bidding equilibrium.

Q.E.D.

Lemma 1.2: *In any symmetric bidding equilibrium, the upper end of the support of equilibrium bids is nonincreasing in x.*

Proof: Suppose there is a symmetric mixed strategy bidding equilibrium and define $\widetilde{B}(x^*)$ to be the upper end of the support of the set of equilibrium bids made by a firm holding x^*. For $\widetilde{B}(x^*)$ to increase in x would require at least one $x^* \in [0,\overline{x}]$, $\widetilde{B}(x^*) > \widetilde{B}(x')\ \forall x' < x^*$. Any firm holding such an x^* cannot win the prize unless it is the largest of all finalists. If a firm holding x^* bids $\widetilde{B}(x^*)$, this implies that if any other firm bids more than $\widetilde{B}(x^*)$, then the firm with x^* cannot win the prize. Therefore, prior to bidding, a firm holding x^* knows it wins the prize with a bid of $\widetilde{B}(x^*)$ *only if* $\widetilde{B}(x^*)$ is the largest of all bids.

Part I. Assume $\Omega(b|\, b^*)$ is differentiable at $\widetilde{B}(x^*)$.

From Lemma 1.1, we know there cannot be a mass point at $\widetilde{B}(x^*)$.

We are looking for the distribution of the *(M - 1)*st largest out of *(N - 1)* bids, but this is just the distribution of the (M - 1)st largest order statistic out of (N - 1) independent bids from the truncated distribution $\Omega(b|\, b^*)$.

This distribution is:[1] Pr[at least {(N - 1) - (M -1) }bids are less than b]

$$= \sum_{i=N-M}^{N} \binom{N}{i} [\Omega(b|b^*)]^i [1-\Omega(b|b^*)]^{N-i}$$

From Lemma 1.1, this probability equals 1 at $b = \widetilde{B}(x^*)$.

$$\left. \frac{\partial\left[\sum_{i=N-M}^{N} \binom{N}{i} [\Omega(b|b^*)]^i [1-\Omega(b|b^*)]^{N-i}\right]}{\partial b} \right|_{b=\widetilde{B}(x^*)} =$$

$$\left. N[\Omega(b|b^*)]^{N-1}\omega(b) \right\} \text{ for } i = N$$

$$\left. \begin{array}{l} + N(N-1)[\Omega(b|b^*)]^{N-2}\omega(b)(0) \\ - N[\Omega(b|b^*)]^{N-1}\omega(b) \end{array} \right\} \text{ for } i = (N-1) \qquad = 0$$

$$+ \{\text{all other terms where N - M} \le i \le N-2 \text{ are zero}\}$$

Therefore, for the firm with x^* that bid $\widetilde{B}(x^*)$, lowering the bid an infinitesimal amount below $\widetilde{B}(x^*)$ has at most a second-order effect on the probability of gaining entry when x^* is the largest x and can win the prize. Hence, lowering the bid has only a second-order effect on the expected earnings of x^* when x^* bids $\widetilde{B}(x^*)$. On the other hand, since the bids are assumed to be differentiable at $\widetilde{B}(x^*)$, then lowering the bid will reduce the firm's probability of entry: $\partial\theta(b^*) / \partial b > 0$. In

[1] Arnold, Balikrishnan, and Nagaraja: A First Course in Order Statistics, pg 12.

other words, lowering the bid has a positive probability of causing the firm to become the *(M+1)st* highest bidder when it would otherwise have been the *M*th highest bidder because it is not the largest x in the contest. Thus, lowering its bid may enable the firm to avoid entry in exactly the circumstances when it doesn't want to pay an entry fee! Additionally, except in the unique case where $\widetilde{B}(x^*)$ is the singularly largest bid and there is zero probability that anyone ever bids more than $\widetilde{B}(x^*)$, then lowering the bid also reduces the expected entry payment. (In discriminatory price auctions lowering the bid *always* reduces the expected entry fee.) Therefore, lowering the bid below $\widetilde{B}(x^*)$ causes a first-order reduction in the firm's expected payments while having at most a second-order effect on the firm's expected earnings. Thus $\widetilde{B}(x^*)$ cannot be an equilibrium bid for a firm holding x^*.

Part II. Suppose that $\Omega(b)$ is not differentiable at $\widetilde{B}(x^*)$, because there is a discrete jump upward in the bids at x^*. This implies that x^* could lower it's bid to be epsilon below the highest bid at the bottom of the interval and the same result goes through as it did in part I, except now the reduction in expected payment is even greater. Therefore, once again we violate our requirements for $\widetilde{B}(x^*) = b^*$ to be an equilibrium bid. Thus, no firm ever has as part of its equilibrium bidding strategy a bid that is strictly greater than all equilibrium bids of all x values below it.

Q.E.D.

Lemma 1.3: *In any symmetric bidding equilibrium, all firms must bid 0.*

Proof: By assumption, the sponsor accepts no negative bids because this only costs the sponsor money and does not raise the expected value of the innovation since any

firm could still win with its initial draw and a bid of zero. The expected profits for a firm holding innovation 0 are:

$$P\Lambda(0,b) - \Gamma(b)\theta(b)$$

Since *F(x)* is continuous, there is zero probability of winning with a draw of 0 and $\Lambda(0,b) = 0$. From lemma 1.2, since the upper end of the support of maximum equilibrium bids is nonincreasing in x the firm would expect a loss with any positive bid since it cannot win the tournament but would have some probability of gaining entry with a positive bid. Therefore, the only possible equilibrium bid by a firm with $x = 0$ is to bid zero. Furthermore, equilibrium bids are continuous at $x = 0$ since the expected profits for a firm holding innovation x are:

$$P\Lambda(x,b) - \Gamma(b)\theta(b)$$

and a bid of $b > 0$ would require:

$$P\Lambda(x,b) - \Gamma(b)\theta(b) \geq P\Lambda(x,0)$$

Rearranging, equilibrium requires:

$$P[\Lambda(x,b) - \Lambda(x,0)] \geq \Gamma(b)\theta(b)$$

As $x \rightarrow 0$, the left hand side converges to zero but the right hand side converges to a strictly positive value for all $b > 0$ if $\theta(b) > 0$. From lemma 1.2, the upper end of the support of bids is nonincreasing which therefore implies $\theta(b) > 0$. Therefore a firm with $x = 0$ bids 0, the bids are continuous at 0 and from lemma 1.2 all other firms must also bid zero. Q.E.D.

Lemma 1.4: *The unique value where $z = y$ is given by the equation:*

$$F(z) = F(y) = 1 - \frac{MC}{P}$$

Proof: To simplify notation define:

$$\mathrm{R}(\cdot) = \left\{\left(\frac{F(x)-F(z)}{1-F(y)}\right) + \left(\frac{F(z)-F(y)}{1-F(y)}\right)\left[F^T(z) + (1-F^T(z))\left(\frac{F(x)-F(z)}{1-F(z)}\right)\right]\right\}$$

$$\mathrm{L}(\cdot) = \left\{\left(\frac{F(z)-F(y)}{1-F(y)}\right)F^T(z)\right\}$$

"Z" is implicitly defined by the equation:

$$P\int_z^u \left\{R^{M-1} - L^{M-1}\right\} f(x)dx - C = 0$$

When $Z = Y$ this expression reduces to:

$$P\int_y^u \left(\frac{F(x)-F(y)}{1-F(y)}\right)^{M-1} f(x)dx - C = 0$$

which can be rewritten as:

$$P[1-F(y)]\int_y^u \left(\frac{F(x)-F(y)}{1-F(y)}\right)^{M-1} \frac{f(x)dx}{1-F(y)} - C = 0$$

Integrating and rearranging, when $z = y$:

$$F(y) = 1 - \frac{MC}{P}.$$

Q.E.D.

Lemma 1.5: $\frac{\partial z}{\partial y} > 0$ **when M = 2.**

Proof:

$$\Lambda(\cdot) = \begin{cases} \left(\frac{F(x)-F(y)}{1-F(y)}\right)F^T(x) & \text{for } y < x < z \\ \left(\frac{F(x)-F(z)}{1-F(y)}\right) + \left(\frac{F(z)-F(y)}{1-F(y)}\right)\left[F^T(z) + (1-F^T(z))\left(\frac{F(x)-F(z)}{1-F(z)}\right)\right] & \text{for } x > z \end{cases}$$

From Taylor's paper we know that z is implicitly defined by the following equation:

$$P\int_{z(y)}^{\bar{x}}\left[\Phi^{M-1}(x;y,z(y),T) - \Phi^{M-1}(z(y);y,z(y),T)\right]f(x)dx - C = 0$$

Substituting for Φ leaves us with:

$$P\int_{z(y)}^{\bar{x}}\left[\left\{\left(\frac{F(x)-F(z)}{1-F(y)}\right) + \left(\frac{F(z)-F(y)}{1-F(y)}\right)\left[F^T(z) + (1-F^T(z))\left(\frac{F(x)-F(z)}{1-F(z)}\right)\right]\right\}^{M-1} - \left\{\left(\frac{F(z)-F(y)}{1-F(y)}\right)F^T(z)\right\}^{M-1}\right]f(x)dx - C = 0$$

For notational simplicity define:

$$G(x;y,z,T) = \left\{\left(\frac{F(x)-F(z)}{1-F(y)}\right) + \left(\frac{F(z)-F(y)}{1-F(y)}\right)\left[F^T(z) + (1-F^T(z))\left(\frac{F(x)-F(z)}{1-F(z)}\right)\right]\right\}$$

$$L(z(y); y, z, T) = \left\{\left(\frac{F(z)-F(y)}{1-F(y)}\right)F^T(z)\right\}$$

Now let us rewrite our expression as:

$$Q(z,y) = P\int_{z(y)}^{\bar{x}}\left[G(x;y,z,T)^{M-1} - L(z;y,z,T)^{M-1}\right]f(x)dx - C = 0$$

First I will show $\frac{\partial Q}{\partial z} < 0$. Which is a requirement for z to be unique.

$$\frac{\partial Q}{\partial z} = P\int_{z(y)}^{\bar{x}}\left[(M-1)G(x;y,z,T)^{M-2}G_z(\cdot) - (M-1)L(z;y,z,T)^{M-2}L_z(\cdot)\right]f(x)dx$$

Note the subscripts refer to partial derivatives.

Therefore, if we can show that $G_z \leq 0$ and $L_z \geq 0$ then we know that $\frac{\partial Q}{\partial z} < 0$.
First we will show that $G_z \leq 0$

Suppose that $x \geq z$ and begin with T = 1.

$$G(T=1) = \left(\frac{F(x)-F(z)}{1-F(y)}\right) + \left(\frac{F(z)-F(y)}{1-F(y)}\right)F(x)$$

Therefore, if T = 1, then:

$$G_Z(T=1) = -\left(\frac{f(z)}{1-F(y)}\right) + \left(\frac{f(z)}{1-F(y)}\right)F(x)$$

Divide by f(z) and multiply by (1-F(y)) to get:

$G_{Z(T=1)} = -1 + F(x) < 0$

Now consider T = S , where S is any arbitrary integer ≥ 0.

The proof that $G_Z < 0$ will follow inductively if we can show that $G_Z(S) > G_Z(S+1)$.

For the general case, G_Z is equal to the following:

$$-\frac{f(z)}{1-F(y)}+\frac{f(z)}{1-F(y)}\left[F^T(z)+(1-F^T(z))\left(\frac{F(x)-F(z)}{1-F(z)}\right)\right]$$

$$+\left(\frac{F(z)-F(y)}{1-F(y)}\right)TF^{T-1}(z)f(z)-\left(\frac{F(z)-F(y)}{1-F(y)}\right)TF^{T-1}(z)f(z)\left(\frac{F(x)-F(z)}{1-F(z)}\right)$$

$$-\left(\frac{F(z)-F(y)}{1-F(y)}\right)\frac{(1-F^T(z))f(z)}{1-F(z)}+\left(\frac{F(z)-F(y)}{1-F(y)}\right)(1-F^T(z))\left(\frac{F(x)-F(z)}{(1-F(z))^2}\right)f(z)$$

Divide by $f(z)$ and multiply by $(1 - F(y))$ to get:

$$-1+F^T(z)+(1-F^T(z))\left(\frac{F(x)-F(z)}{1-F(z)}\right)+TF^{T-1}(z)\big(F(z)-F(y)\big)$$

$$-\big(F(z)-F(y)\big)TF^{T-1}(z)\left(\frac{F(x)-F(z)}{1-F(z)}\right)-\big(F(z)-F(y)\big)\frac{(1-F^T(z))}{1-F(z)}$$

$$+(1-F^T(z))\left(\frac{F(x)-F(z)}{(1-F(z))^2}\right)\big(F(z)-F(y)\big)$$

Now divide by $\left(\frac{F(x)-F(z)}{1-F(z)}\right)$ to get:

$$-(1-F^T(z))\left(\frac{1-F(z)}{F(x)-F(z)}\right)+(1-F^T(z))+TF^{T-1}(z)\big(F(z)-F(y)\big)\left(\frac{1-F(z)}{F(x)-F(z)}\right)$$

$$-\big(F(z)-F(y)\big)TF^{T-1}(z)-\left(\frac{F(z)-F(y)}{F(x)-F(z)}\right)\big(1-F^T(z)\big)+(1-F^T(z))\left(\frac{F(z)-F(y)}{1-F(z)}\right)$$

Now divide by (F(z) - F(y)) and collect terms to get:

[Note we are *assuming* here that $Z > y$ so that this does not change the sign]

$$\left\{\left(\frac{(1-F^T(z))}{F(z)-F(y)}\right)-TF^{T-1}(z)\right\}\left[1-\left(\frac{1-F(z)}{F(x)-F(z)}\right)\right]+(1-F^T(z))\left[\left(\frac{1}{1-F(z)}\right)-\left(\frac{1}{F(x)-F(z)}\right)\right]$$

So to show that $G_Z(S) \geq G_Z(S+1)$ we need to show that:

$$\left\{\left(\frac{(1-F^S(z))}{F(z)-F(y)}\right)-SF^{S-1}(z)\right\}\left[1-\left(\frac{1-F(z)}{F(x)-F(z)}\right)\right]+(1-F^S(z))\left[\left(\frac{1}{1-F(z)}\right)-\left(\frac{1}{F(x)-F(z)}\right)\right]$$

$$\geq\left\{\left(\frac{(1-F^{S+1}(z))}{F(z)-F(y)}\right)-SF^{S}(z)\right\}\left[1-\left(\frac{1-F(z)}{F(x)-F(z)}\right)\right]+(1-F^{S+1}(z))\left[\left(\frac{1}{1-F(z)}\right)-\left(\frac{1}{F(x)-F(z)}\right)\right]$$

The last expression in each equation is negative. Therefore, since

$(1-F^{S+1}(z)) > (1-F^S(z))$

the last expression on the right hand side is more negative than the last expression on the left hand side. Thus, all we need to show is:

$$\left\{\left(\frac{(1-F^S(z))}{F(z)-F(y)}\right)-SF^{S-1}(z)\right\}\left[1-\left(\frac{1-F(z)}{F(x)-F(z)}\right)\right] \geq$$

$$\left\{\left(\frac{(1-F^{S+1}(z))}{F(z)-F(y)}\right)-SF^{S}(z)\right\}\left[1-\left(\frac{1-F(z)}{F(x)-F(z)}\right)\right]$$

Now factor out the term $\left[1-\left(\frac{1-F(z)}{F(x)-F(z)}\right)\right]$ which is *negative* so we need to show:

$$-F^S(z)-SF^{S-1}(z)\big(F(z)-F(y)\big)\leq -F^{S+1}(z)-SF^S(z)\big(F(z)-F(y)\big)$$

Divide by $F^{S-1}(z)$ and rearrange terms so that we need to show:

$$F^2(z)+SF(z)\big(F(z)-F(y)\big)\leq F(z)+S\big(F(z)-F(y)\big)$$

Which is obviously true.

We have shown that $G_Z(T=1) < 0$, and we have shown that $G_Z(T=S) > G_Z(T=S+1)$,

so by induction we have proven that for all T, $G_Z < 0$.

Now to finish our proof that $\frac{\partial Q}{\partial z} < 0$ we also need to show that $L_Z \geq 0$.

Note that $L_Z = \left(\frac{F^T(z)}{1-F(y)}\right)f(z) + T\left(\frac{F(z)-F(y)}{1-F(y)}\right)F^{T-1}(z)f(z)$

Now all terms are greater than or equal to zero for any value of T therefore $L_Z \geq 0$.

Thus we have shown that $\frac{\partial Q}{\partial z} < 0$.

For part 2 of determining the sign of dz/dy, we need to determine the sign of dQ/dy when $M = 2$. Holding Z constant we differentiate with respect to y.

$$\frac{\partial Q}{\partial y} = P\int_z^{\bar{x}}\left[(M-1)G(x;y,z,T)^{M-2}G_y(\cdot) - (M-1)L(z;y,z,T)^{M-2}L_y(\cdot)\right]f(x)dx$$

When M = 2, this reduces to:

$$\frac{\partial Q}{\partial y} = P(M-1)\int_z^{\bar{x}}\left[G_y(\cdot) - L_y(\cdot)\right]f(x)dx$$

$$L_y = -\left(\frac{1}{1-F(y)}\right)\left(\frac{1-F(z)}{1-F(y)}\right)F^T(z)$$

$$G_y = -\left(\frac{1}{1-F(y)}\right)\left(\frac{1-F(x)}{1-F(y)}\right)F^T(z)$$

Since by assumption $x \geq z$, sign of $\frac{\partial Q}{\partial y}$ = sign of $[(1-F(z)) - (1-F(x))] \geq 0$

Finally, $\dfrac{\partial z}{\partial y} = \dfrac{-\partial Q/\partial y}{\partial Q/\partial z} \geq 0$ when $M = 2$

Therefore, with only two contestants, z is an increasing function of y. Q.E.D.

Demonstration of No Pure-Strategy Equilibrium in tournaments with research:

$$T > 0$$

Define the following additional terms:

x_1 = the initial innovation held by a firm prior to the tournament.

x_T = the best draw realized by a firm in all periods following entry.

C = the *expected* cost of research given entry

Proposition 1.1: *Entry auctions are inefficient mechanisms for selecting contestants in multiple-period research tournaments when contestants have different initial endowments.*

Proof: Suppose there is a strictly increasing pure-strategy equilibrium.

A firm's expected profit from an endowment of x_1* and a bid of $b*$ is:

$$P\Lambda(x_1*, b*) - \Gamma(b*)\theta(b*) - C\theta(b*)$$

We can partition $\Lambda(x_1*, b*)$ into the probability of winning with one's initial draw and the probability of winning with a draw from a period following entry:

$$\Lambda(x_1*, b*) = \xi(x_1*, b*) + \xi(x_T, b*|x_1*)$$

where $\xi(x_1*, b*)$ = the joint probability of gaining entry and winning with x_1*.

$\xi(x_T, b*|x_1*)$ = the conditional probability of winning with a draw, $x_T > x_1*$

Subsequent draws are only made if x_1* was less than the realized z-stop, and can only win if they are larger than x_1*. Thus, a firm's expected profits can be written:

$$P\left[\xi(x_1{*},b{*}) \;+\; \xi(x_{\mathrm{T}},b{*}|x_1{*})\right] - \Gamma(b)\theta(b) - C\theta(b)$$

For any player holding an initial draw which exceeds the *z-stop* value revealed after entry, this player will not conduct research following entry and therefore $\xi(x_{\mathrm{T}}, b{*}|x_1{*}) = 0$ and will incur no costs of research. As long as $\mathrm{P} < \infty$ and $\mathrm{C} > 0$, then the maximum possible value for z will be strictly less than $\bar{x}$ so there is always an interval of x that will not conduct research following entry. Thus, the expected profit of a firm holding one of these draws is:

$$P\xi(x_1{*},b{*}) - \Gamma(b{*})\theta(b{*})$$

Since bids are assumed strictly increasing, the expected profits are (dropping the subscripts):

$$\begin{aligned}
&P\int_0^{x*}\left(\frac{F(x*)-F(y)}{1-F(y)}\right)^{M-1} \\
&\times \left\{\sum_{i=0}^{M-1}\binom{M-1}{i}\left(\frac{F(z(y))-F(y)}{F(x*)-F(y)}\right)^{i}\left(\frac{F(x*)-F(z(y))}{F(x*)-F(y)}\right)^{M-1-i}\right. \\
&\times \left.\left[F(z(y))^{T} + \left(1-F(z(y))^{T}\right)\left(\frac{F(x*)-F(z(y))}{1-F(z(y))}\right)\right]^{i}\right\} h_{m:n}(y)dy \\
&- \int_0^{x*} B(y)h_{m:n}(y)dy
\end{aligned}$$

Appealing to Lemma A1, equilibrium requires $\pi_1(\hat{x},x*) = 0$ evaluated at $\hat{x} = x*$

$$\pi_1(\hat{x},x^*) = P\left(\frac{F(x^*)-F(\hat{x})}{1-F(\hat{x})}\right)^{M-1}$$

$$\times \left\{\sum_{i=0}^{M-1}\binom{M-1}{i}\left(\frac{F(z(\hat{x}))-F(\hat{x})}{F(x^*)-F(\hat{x})}\right)^i\left(\frac{F(\bar{x})-F(z(\hat{x}))}{F(x^*)-F(\hat{x})}\right)^{M-1-i}\right.$$

$$\left.\times \left[F(z(\hat{x}))^T + \left(1-F(z(\hat{x}))^T\right)\left(\frac{F(x^*)-F(z(\hat{x}))}{1-F(z(\hat{x}))}\right)\right]^i\right\}h_{m:n}(\hat{x})$$

$$- \ B(\hat{x})h_{m:n}(\hat{x})$$

Therefore, at $\hat{x} = x^*$ setting $\pi_1(\hat{x},x^*) = 0$ gives us:

$\pi_1(x^*,x^*) = - \ B(x^*)h_{m:n}(x^*) = 0 \Rightarrow B(x^*) = 0.$ Q.E.D.

Notice that this is consistent with the bids we had when T = 0, which makes sense because for values of x that are very large they do not do any extra research following entry.

Restoring equilibrium to the entry auction.

Uniform-Price Auction

Returning to our tournament without research, T = 0. Suppose a firm of type x_i chooses to deviate from the equilibrium and bid as type $r < x_i$, then its expected profits are:

(1) $$\pi(r,x_i) = P(B(r))\int_0^r \left(\frac{F(x_i)-F(y)}{1-F(y)}\right)^{M-1} h_{m:n}(y)dy - \int_0^r B(y)h_{m:n}(y)dy$$

This time the prize is a function of the bid, which in equilibrium depends on the type you *say* you are. If a firm of type x_i chooses to deviate from the equilibrium strategy and bid as type $r > x_i$ its expected profits are:

$$(2) \qquad \pi(r,x_i) = P(B(r))\int_0^{x_i} \left(\frac{F(x_i)-F(y)}{1-F(y)}\right)^{M-1} h_{m:n}(y)dy - \int_0^r B(y)h_{m:n}(y)dy$$

If $B(x)$ is an equilibrium bidding function, then $B^{-1}(b) = x_i$ must maximize firm i's expected profits. The first order conditions for a maximum require $\pi_1(r,x_i)\big|_{r=x_i}=0$

Checking the first order conditions for (1) and (2) gives us:

$$(1a) \qquad \begin{aligned}\pi_1(r,x_i) &= P'(b)B'(r)\int_0^r \left(\frac{F(x_i)-F(y)}{1-F(y)}\right)^{M-1} h_{m:n}(y)dy \\ &+ P(B(r))\left(\frac{F(x_i)-F(r)}{1-F(r)}\right)^{M-1} h_{m:n}(r) - B(r)h_{m:n}(r)\end{aligned}$$

$$(2a) \qquad \pi_1(r,x_i) = P'(b)B'(r)\int_0^{x_i} \left(\frac{F(x_i)-F(y)}{1-F(y)}\right)^{M-1} h_{m:n}(y)dy - B(r)h_{m:n}(r)$$

Letting $r = x_i$ and setting equal to zero, both equations leave us with:

$$\pi_1(x_i,x_i) = P'(b)B'(x_i)\int_0^{x_i} \left(\frac{F(x_i)-F(y)}{1-F(y)}\right)^{M-1} h_{m:n}(y)dy - B(x_i)h_{m:n}(x_i) = 0$$

Integrating repeatedly by parts, the first term is just equal to:

$$P'(b)B'(x_i)F(x_i)^N$$

To see this note that:

$$\int_0^{x_i} \left(\frac{F(x_i)-F(y)}{1-F(y)}\right)^{M-1} h_{m:n}(y)dy = \int_0^{x_i} \frac{N!}{(N-M)!(M-1)!}[F(y)]^{N-M}\left(F(x_i)-F(y)\right)^{M-1} f(y)dy$$

We will integrate repeatedly by parts:

Let u = $[F(x)-F(y)]^{M-1}$ and du = $-(M-1)[F(x)-F(y)]^{M-2} f(y)$

Let dv = $[F(y)]^{N-M} f(y)$ and v = $\dfrac{1}{N-M+1}[F(y)]^{N-M+1}$

Integrating by parts we know: $\int_0^x udv = uv\,\big|_0^x - \int_0^x vdu$

Therefore, we have:

$$\frac{N!}{(N-M)!(M-1)!}\left.\frac{[F(x)-F(y)]^{M-1}[F(y)]^{N-M+1}}{N-M+1}\right|_{o}^{x}$$

$$+\int_0^x \frac{N!}{(N-M+1)!(M-2)!}[F(y)]^{N-M+1}[F(x)-F(y)]^{M-2} f(y)dy$$

But the first term goes to zero so we have:

$$\int_0^x \frac{N!}{(N-M+1)!(M-2)!}[F(y)]^{N-M+1}[F(x)-F(y)]^{M-2} f(y)\,dy$$

This can similarly be integrated by parts doing the same kind of thing which leaves:

$$\int_0^x \frac{N!}{(N-M+2)!(M-3)!}[F(y)]^{N-M+2}[F(x)-F(y)]^{M-3} f(y)\,dy$$

This is repeated until the $M-i$ term goes to zero, and the $N-M+j$ term goes to $N-1$, at that time we are left with:

$$\int_0^x N[F(y)]^{N-1} f(y)\,dy$$

And this easily integrates out to be: $[F(x_i)]^N$

Therefore, returning to our original equation, the equilibrium bid is:

$$P'(b)B'(x_i)F(x_i)^N = B(x)h_{m:n}(x)$$

or

$$B'(x_i) = \frac{B(x_i)h_{m:n}(x_i)}{P'(b)F(x_i)^N}$$

Which simply says that bids will be increasing in x so long as the prize is increasing in the bids. According to Lemma A1, we are only halfway done with the proof. We have shown above that with this bidding strategy: $\forall x \quad \pi_1(x,x) = 0$. However, it remains to be shown that: $\forall x$, $\forall r \quad \pi_{12}(r,x) \geq 0$ so we will do that now. In the case where $r < x$, we have:

$$\pi_{12}(r,x_i) = P'(b)B'(r)\int_0^r (M-1)\left(\frac{F(x_i)-F(y)}{1-F(y)}\right)^{M-2} f(x_i)h_{m:n}(y)dy$$
$$+ \quad P(B(r))(M-1)\left(\frac{F(x_i)-F(r)}{1-F(r)}\right)^{M-2} f(x_i)h_{m:n}(r)$$

In the case where $r > x$, we have:

$$\pi_{12}(r,x_i) = P'(b)B'(r)\int_0^{x_i} (M-1)\left(\frac{F(x_i)-F(y)}{1-F(y)}\right)^{M-2} f(x_i)h_{m:n}(y)dy$$

Both of these equations are greater than or equal to zero as long as P'(b) is greater than or equal to zero. Therefore, if we make the prize an increasing function of the bid, then we fulfill both requirements for a pure-strategy equilibrium bidding function.

Solving for an equilibrium:

Any equilibrium bidding function must satisfy the following equation:

$$P'(x) = \frac{B(x)h_{m:n}(x)}{F(x_i)^N}$$

However we know that:

$$h_{m:n}(x) = \frac{n!}{(n-m)!(m-1)!}\left[F(x)\right]^{n-m}\left[1-F(x)\right]^{m-1} f(x)$$

Substituting:

$$P'(x) = \frac{B(x)\frac{n!}{(n-m)!(m-1)!}\left[F(x)\right]^{n-m}\left[1-F(x)\right]^{m-1} f(x)}{F(x_i)^N}$$

Letting $K = \dfrac{n!}{(n-m)!(m-1)!}$ and reducing we have the simple equation:

$$P'(x) = \frac{B(x)K\left[1-F(x)\right]^{m-1} f(x)}{F(x_i)^m}$$

One solution that works for all distributions is as follows:

Let $B(x) = F(x)^m$ and $P'(x) = K[1-F(x)]^{m-1} f(x)$.

So, in particular, $B(0) = 0$ and the bids are strictly increasing in x.

This makes $P'(x) \geq 0$.

Then $P(x) = C - \dfrac{K}{M}[1-F(x)]^M$ where C is a *constant from integration*.

From our equilibrium bidding function we know that $x = F^{-1}\left[b^{1/M}\right]$.

Substituting this back into the equation we get: $P(b) = C - \dfrac{K}{M}\left[1 - b^{1/M}\right]^M$

Now we want to choose C so that $P(0) = 0$ for this tournament. Therefore we set

$$C = \frac{K}{M}.$$

So, we have a solution of:

$$P(b) = \frac{K}{M} - \frac{K}{M}\left[1 - b^{1/M}\right]^{M} \quad \text{and} \quad B(x) = F(x)^{m}$$

This solution satisfies our criteria of having bids be strictly increasing in x.

However, we can choose any arbitrary positive number, L, and have another equilibrium:

$$P(b) = \frac{K}{LM} - \frac{K}{LM}\left[1 - (Lb)^{1/M}\right]^{M} \quad \text{and} \quad B(x) = \frac{F(x)^{m}}{L}$$

As $L \to \infty$, the prize payments and the bids converge to zero!

Finally, notice that for specific distributions, we can find other different forms of functional equations that also satisfy our first order equilibrium conditions.

Suppose that $x \sim U[0,1]$ so $f(x) = 1$ and $f'(x) = 0$.

Our first order conditions would be: $P'(x) = \dfrac{B(x)K[1-x]^{m-1}}{x^{m}}$

Let $B(x) = x^{2M-1}$. Therefore, $P'(x) = K(x - x^{2})^{M-1}$.

This implies that: $P(x) = K\left(\dfrac{x^{2}}{2} - \dfrac{x^{3}}{3}\right)$ and $x = b^{1/(2M-1)}$.

Substituting: $P(b) = K\left(\dfrac{b^{2/(2M-1)}}{2} - \dfrac{b^{3/(2M-1)}}{3}\right)$ and $B(x) = x^{2M-1}$

This solution still has bids monotonically increasing in x, starting with B(0) = 0. Moreover, P(0) = 0, and increases as the bidding increases.

Discriminatory Auction

Following the same analysis as the uniform-price auction, when the prize is dependent upon the bid, we get the following result for the discriminatory auction:

$$B(x_i) = \frac{P'(b)B'(x_i)F(x_i)^N}{f_{m:n}(x_i)} - \frac{B'(x_i)F_{m:n}(x_i)}{f_{m:n}(x_i)}$$

This result is quite similar, the only difference is that the bids are slightly lower because now there is a direct cost to bidding instead of the "indirect" cost from a uniform auction.

Lemmas A2, A3, and A4 are an assortment of interesting propositions, not required in the paper, but which shed insight into the problems associated with establishing a symmetric bidding equilibrium for the entry auction.

Lemma A2: *Any continuous pure-strategy bidding function, B(x), that exists for the tournament without research (i.e., T = 0), must be non-decreasing in x.*

Proof: Suppose not. Suppose that $x_1 > x_2$ but $b_2 > b_1$.

$b_2 > b_1$ implies $\theta(b_2) > \theta(b_1)$, similarly $x_1 > x_2$ implies $F(x_1) > F(x_2)$.

Incentive Compatibility constraints require:

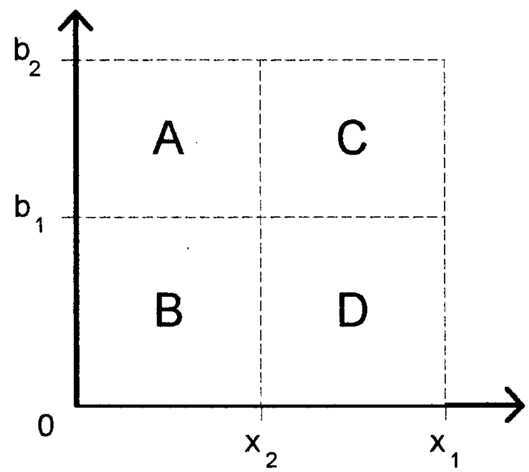
I.C. Firm 1: $P\Lambda(x_1, b_1) - \Gamma(b_1)\theta(b_1) \geq P\Lambda(x_1, b_2) - \Gamma(b_2)\theta(b_2)$

I.C. Firm 2: $P\Lambda(x_2, b_2) - \Gamma(b_2)\theta(b_2) \geq P\Lambda(x_2, b_1) - \Gamma(b_1)\theta(b_1)$

Adding together, canceling terms and rearranging -- incentive compatibility requires:

$$\Lambda(x_1,b_1)+\Lambda(x_2,b_2)-\Lambda(x_1,b_2)-\Lambda(x_2,b_1)\geq 0$$

Graphically:



where:

$\Lambda(x_1,b_1)$ = B + D

$\Lambda(x_2,b_2)$ = A + B

$\Lambda(x_1,b_2)$ = A + B + C + D

$\Lambda(x_2,b_1)$ = B

$\Lambda(x_1,b_1)+\Lambda(x_2,b_2)-\Lambda(x_1,b_2)-\Lambda(x_2,b_1)\geq 0$ is equivalent to:

$$\int_0^{b_1}\int_0^{x_1}\lambda(x,b)dxdb+\int_0^{b_2}\int_0^{x_2}\lambda(x,b)dxdb-\int_0^{b_2}\int_0^{x_1}\lambda(x,b)dxdb-\int_0^{b_1}\int_0^{x_2}\lambda(x,b)dxdb\geq 0$$

Subtracting "like" terms we get:

$$\int_0^{b_1}\int_{x_2}^{x_1}\lambda(x,b)dxdb+\int_0^{b_2}\int_{x_1}^{x_2}\lambda(x,b)dxdb\geq 0$$

Reducing:

$$-\int_{b_1}^{b_2}\int_{x_2}^{x_1}\lambda(x,b)dxdb\geq 0$$

Which is a contradiction if we assume the bidding equilibrium is continuous so that λ has positive density everywhere. Q.E.D.

Lemma A3: *If there is a symmetric mixed strategy equilibrium, then different values of x can share only a single bid in common.*

Proof: $\pi(x,b) = P\Lambda(x,b) - \Gamma(b)\theta(b)$

Suppose two possible bids are: b and $(b - \varepsilon)$. Let us define:

$$\theta(b-\varepsilon) = \theta(b) - \Delta\theta(\varepsilon)$$
$$\Gamma(b-\varepsilon) = \Gamma(b) - \Delta\Gamma(\varepsilon)$$
$$\Lambda(x,b-\varepsilon) = \Lambda(x,b) - \Delta\Lambda(x,\varepsilon)$$

If b and $(b - \varepsilon)$ are two bids that a contestant holding x randomizes over, they must have the same expected payoff, say K, in equilibrium:

$$\pi(x,b) = P\Lambda(x,b) - \Gamma(b)\theta(b) = K$$
$$\pi(x,b-\varepsilon) = P\Lambda(x,b-\varepsilon) - \Gamma(b-\varepsilon)\theta(b-\varepsilon) = K$$

Therefore, we can set them equal to each other:

$$P\Lambda(x,b) - \Gamma(b)\theta(b) = P\Lambda(x,b-\varepsilon) - \Gamma(b-\varepsilon)\theta(b-\varepsilon)$$

Expanding the probabilities we get:

$$P\Lambda(x,b) - \Gamma(b)\theta(b) = P[\Lambda(x,b) - \Delta\Lambda(x,\varepsilon)] - \Gamma(b)[\theta(b) - \Delta\theta(\varepsilon)] + \Delta\Gamma(\varepsilon)\theta(b-\varepsilon)$$

Cancelling similar terms leaves us with:

$$0 = -P\Delta\Lambda(x,\varepsilon) + \Gamma(b)\Delta\theta(\varepsilon) + \Delta\Gamma(\varepsilon)\theta(b-\varepsilon)$$

Suppose there were two values of x: $x_1 > x_2$ and two of the bids these values randomized were the same. Then it must be that:

$$0 = -P\Delta\Lambda(x_1,\varepsilon) + \Gamma(b)\Delta\theta(\varepsilon) + \Delta\Gamma(\varepsilon)\theta(b-\varepsilon)$$
$$0 = -P\Delta\Lambda(x_2,\varepsilon) + \Gamma(b)\Delta\theta(\varepsilon) + \Delta\Gamma(\varepsilon)\theta(b-\varepsilon)$$

But this implies:

$$\Delta\Lambda(x_1,\varepsilon) = \Delta\Lambda(x_2,\varepsilon)$$

Which is a contradiction because $x_1 > x_2$. Thus, we know that if there is a mixed strategy then different values of x can only share a single bid in common. Since x values are continuous, this rules out mixed strategies which randomize over intervals, so any symmetric mixed strategy must randomize over discrete bids -- if one exists.

Lemma A4: *Any mixed strategy equilibrium cannot have a differentiable interior.*

Proof: Suppose $\forall$x support $\mathrm{S(x)} = [\underline{b}, \bar{b}]$.

Equilibrium requires: $\dfrac{\partial \Lambda(b,x)}{\partial b} > 0$ or everyone would bid 0 which isn't an equilibrium.

A differentiable mixed strategy equilibrium requires: $\dfrac{\partial \pi(b,x)}{\partial b} \equiv 0$

This implies: $P\dfrac{\partial \Lambda(b,x)}{\partial b} = \dfrac{\partial \{\Gamma(b)\theta(b)\}}{\partial b}$

But, this is not feasible because $P\dfrac{\partial^2 \Lambda(b,x)}{\partial b \partial x} \neq 0$ but $\dfrac{\partial[\Gamma(b)\theta(b)]}{\partial b \partial x} = 0$.

Hence any mixed strategy equilibrium cannot have a differentiable interior.

Appendix 1B

Demonstration of Bid Inversion

The following pages formulate the expected profits of a firm if we assume that bids are strictly increasing in equilibrium and the tournament has a fixed prize. This section demonstrates that when M = 2 and T = 1 if bids were strictly increasing and the prize was a fixed size we would get a tendency for bid inversions. Of course, this is not an equilibrium, but is only illustrative of the problems which are caused by allowing research following entry. In essence, this section verifies Proposition 1.1 for the case when M = 2 and T = 1.

ASSUME there is a pure-strategy bidding equilibrium that is monotonically increasing in x.

$$\sum_{i=0}^{M-1}\binom{M-1}{i}\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)^i\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)^{M-1-i}[F(x_1)]^i =$$

the probability that i of the other (M-1) entrants drew less than z (given that they were less than x_1 and better than y), multiplied by the probability their 2nd draw is less than x_1.

$$\int_0^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^{M-1}\left\{\sum_{i=0}^{M-1}\binom{M-1}{i}\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)^i\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)^{M-1-i}[F(x_1)]^i\right\}h_{m:n}(y)dy$$

is the probability of winning with your first draw when it is larger than z so you do not make a second draw even though the others might if their first was less than z. Note above that the limits of integration go from 0 to $z^{-1}(x_1)$. This ensures that

y was small enough so that $x_1 > z(y)$. Remember when M = 2, z(y) has an inverse. Note that if $x_1 < z(0)$ we *define* $z^{-1}(x_1) = 0$.

$$\int_{z^{-1}(x_1)}^{x_1} \left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^{M-1} [F(x_1)]^M h_{m,n}(y)dy$$

is the probability of winning with x_1 when it is smaller than z. Therefore, all other entrants must have also had endowments smaller than z.

$$\int_{z^{-1}(x_1)}^{x_1} \left[\int_{x_1}^{z(y)} \left(\frac{F(x_2)-F(y)}{1-F(y)}\right)^{M-1} (F(x_2))^{M-1} f(x_2)dx_2\right] h_{m,n}(y)dy$$

is the probability of winning with your second draw, x_2. This requires your first draw to be smaller than z(y) -- which defines the limit of dy. It also requires your second draw be larger than x_1 giving the lower bound of x_2. Your second draw must be larger than all other M-1 entrants' first draws and in this case the second draw is also smaller than z(y) so that all M-1 other entrants conduct research and their second draw must also be less than x_2.

$$\int_{z^{-1}(x_1)}^{x_1} \left[\int_{z(y)}^{\bar{x}} \left(\frac{F(x_2)-F(y)}{1-F(y)}\right)^{M-1}\right.$$

$$\left.\times \left\{\sum_{i=0}^{M-1} \binom{M-1}{i} \left(\frac{F(z(y))-F(y)}{F(x_2)-F(y)}\right)^i \left(\frac{F(x_2)-F(z(y))}{F(x_2)-F(y)}\right)^{M-1-i} [F(x_2)]^i\right\} f(x_2)dx_2\right] h_{m,n}(y)dy$$

This is the probability that you win with your second draw, but that it is larger than z(y) which means that some of the other entrants may not have made a second draw even though you did. Notice we could not just use this equation by itself and run the limits of x_2 from x_1 to $\bar{x}$ because then at some points we would have had z(y) > x_2 which would have made the second binomial probability term > 1.

EXPECTED PROFIT IN A TOURNAMENT WITH T = 1 AND M = 2.

(Substituting for M = 2)

$$P\int_{0}^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)\left\{\sum_{i=0}^{1}\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)^{i}\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)^{1-i}\left[F(x_1)\right]^{i}\right\}h_{m:n}(y)dy$$

$$+\ P\int_{z^{-1}(x_1)}^{x_1}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)\left[F(x_1)\right]^{2}h_{m:n}(y)dy$$

$$+\ P\int_{z^{-1}(x_1)}^{x_1}\left[\int_{x_1}^{z(y)}\left(\frac{F(x_2)-F(y)}{1-F(y)}\right)F(x_2)f(x_2)dx_2\right]h_{m:n}(y)dy$$

$$+P\int_{z^{-1}(x_1)}^{x_1}\int_{z(y)}^{\bar{x}}\left(\frac{F(x_2)-F(y)}{1-F(y)}\right)$$

$$\times\left\{\sum_{i=0}^{1}\left(\frac{F(z(y))-F(y)}{F(x_2)-F(y)}\right)^{i}\left(\frac{F(x_2)-F(z(y))}{F(x_2)-F(y)}\right)^{1-i}\left[F(x_2)\right]^{i}\right\}f(x_2)dx_2h_{m:n}(y)dy$$

$$-\int_{0}^{x_1}B(y)h_{m:n}(y)dy\ -\int_{z^{-1}(x_1)}^{x_1}C\,h_{m:n}(y)dy$$

Suppose the monotonic, pure-strategy bidding function is $B(x)=b$. Any incentive compatible equilibrium bidding function must satisfy the requirement:

$\pi(b_1,x_1)=\pi(x_1,x_1)\geq\pi(\hat{x}_1,x_1)\quad\forall\,\hat{x}_1,x_1$ where $\hat{x}_1$ implicitly refers to the bid.

$\pi(\hat{x}_1,x_1)=$

$$P\int_0^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)\left\{\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)+\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)F(x_1)\right\}h_{m:n}(y)dy$$

$$+\;P\int_{z^{-1}(x_1)}^{\hat{x}_1}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)[F(x_1)]^2\,h_{m:n}(y)dy$$

$$+\;P\int_{z^{-1}(x_1)}^{\hat{x}_1}\left[\int_{x_1}^{z(y)}\left(\frac{F(x_2)-F(y)}{1-F(y)}\right)F(x_2)f(x_2)dx_2\right]h_{m:n}(y)dy$$

$$+P\int_{z^{-1}(x)}^{\hat{x}_1}\int_{z(y)}^{\bar{x}}\left(\frac{F(x_2)-F(y)}{1-F(y)}\right)$$

$$\times\left\{\left(\frac{F(x_2)-F(z(y))}{F(x_2)-F(y)}\right)+\left(\frac{F(z(y))-F(y)}{F(x_2)-F(y)}\right)F(x_2)\right\}f(x_2)dx_2h_{m:n}(y)dy$$

$$-\int_0^{\hat{x}_1}\mathrm{B}(y)h_{m:n}(y)dy\;-\;\int_{z^{-1}(x_1)}^{\hat{x}_1}C\,h_{m:n}(y)dy$$

The foc's for a maximum require: $\pi_1(\hat{x}_1, x_1) = 0$ when evaluated at $\hat{x}_1 = x_1$.

Taking the derivative:

$$\pi_1(\hat{x}_1, x_1) =$$

$$P\left(\frac{F(x_1) - F(\hat{x}_1)}{1 - F(\hat{x}_1)}\right)[F(x_1)]^2 h_{m:n}(\hat{x}_1)$$

$$+ \quad P\left[\int_{x_1}^{z(\hat{x}_1)} \left(\frac{F(x_2) - F(\hat{x}_1)}{1 - F(\hat{x}_1)}\right) F(x_2) f(x_2) dx_2 \right] h_{m:n}(\hat{x}_1)$$

$$+P \int_{z(\hat{x}_1)}^{\bar{x}} \left(\frac{F(x_2) - F(\hat{x}_1)}{1 - F(\hat{x}_1)}\right) \left\{ \left(\frac{F(x_2) - F(z(\hat{x}_1))}{F(x_2) - F(\hat{x}_1)}\right) + \left(\frac{F(z(\hat{x}_1)) - F(\hat{x}_1)}{F(x_2) - F(\hat{x}_1)}\right) F(x_2) \right\} f(x_2) dx_2 h_{m:n}(\hat{x}_1)$$

$$- \mathrm{B}(\hat{x}_1) h_{m:n}(\hat{x}_1) - C h_{m:n}(\hat{x}_1)$$

Evaluating at $\hat{x}_1 = x_1$ and then setting equal to zero leaves us with:

$$\mathrm{B}(x_1) = P \int_{x_1}^{z(x_1)} \left(\frac{F(x_2) - F(x_1)}{1 - F(x_1)}\right) F(x_2) f(x_2) dx_2$$

$$+P \int_{z(x_1)}^{\bar{x}} \left(\frac{F(x_2) - F(z(x_1))}{1 - F(x_1)}\right) f(x_2) dx_2$$

$$+P \int_{z(x_1)}^{\bar{x}} \left(\frac{F(z(x_1)) - F(x_1)}{1 - F(x_1)}\right) F(x_2) f(x_2) dx_2$$

$$- C$$

Suppose that this is a pure-strategy equilibrium bidding function.

What are its' properties?

$$\frac{\partial B(x_1)}{\partial x_1} =$$

$$= P\left(\frac{F(z(x_1)) - F(x_1)}{1 - F(x_1)}\right) F(z(x_1)) f(z) z'(x_1) \ + \ P \int_{x_1}^{z(x_1)} \frac{[F(x_2) - 1]}{[1 - F(x_1)]^2} f(x_1) F(x_2) f(x_2) dx_2$$

$$+ \ P \int_{z(x_1)}^{\bar{x}} \left(\frac{F(x_2) - F(z(x_1))}{[1 - F(x_1)]^2}\right) f(x_1) f(x_2) dx_2 \ - \ P \int_{z(x_1)}^{\bar{x}} \frac{f(z) z'(x_1)}{1 - F(x_1)} f(x_2) dx_2$$

$$- \ P\left(\frac{F(z(x_1)) - F(x_1)}{1 - F(x_1)}\right) F(z(x_1)) f(z) z'(x_1) \ + \ P \int_{z(x_1)}^{\bar{x}} \frac{f(z) z'(x_1)}{1 - F(x_1)} f(x_2) F(x_2) dx_2$$

$$- \ P \int_{z(x_1)}^{\bar{x}} \frac{f(x_1)}{[1 - F(x_1)]} F(x_2) f(x_2) dx_2 \ + \ P \int_{z(x_1)}^{\bar{x}} \left(\frac{F(z(x_1)) - F(x_1)}{[1 - F(x_1)]^2}\right) f(x_1) F(x_2) f(x_2) dx_2$$

The first and fifth terms cancel out. Then, we combine the fourth and sixth terms.

Combine the rest over a common denominator and cancel out terms to be left with:

$$\frac{\partial B(x_1)}{\partial x_1} = \ -P \int_{x_1}^{z(x_1)} \frac{[1 - F(x_2)][F(x_2) + F(z(x_1))]}{[1 - F(x_1)]^2} f(x_1) f(x_2) dx_2$$

$$- \ P \int_{z(x_1)}^{\bar{x}} \frac{[1 - F(x_2)]}{[1 - F(x_1)]} f(z) z'(x_1) f(x_2) dx_2$$

Which is negative! So if this were an equilibrium bidding function, it would be <u>*decreasing*</u> in the initial draws! But our hypothesis which these equations are based upon assumes the equilibrium bidding function is *increasing* in each players initial draw. So there cannot be a pure-strategy equilibrium which is strictly increasing.

EXPECTED PROFITS WITH ANY M AND T

$$\int_{z^{-1}(x_1)}^{x_1}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^{M-1}\left[F(x_1)\right]^{MT}h_{m:n}(y)dy$$

Above is the probability of winning with your endowment when it is less than z so every entrant makes all subsequent draws.

$$\int_{0}^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^{M-1}\left\{\sum_{i=0}^{M-1}\binom{M-1}{i}\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)^{i}\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)^{M-1-i}\right.$$
$$\left.\times\left[F(z(y))^{T}+\left(1-F(z(y))^{T}\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]^{i}\right\}h_{m:n}(y)dy$$

Above is the probability of winning with endowment when it is larger than z so you do not make other draws but other contestants may make additional draws.

$$\int_{z^{-1}(x_1)}^{x_1}\left[\int_{x_1}^{z(y)}\left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^{M-1}\left[F(x_T)\right]^{T(M-1)}f_{1:T}(x)dx_T\right]h_{m:n}(y)dy$$

Above is the probability of winning with a subsequent draw which is less than z. Note that this also requires your endowment to be less than z, in addition every player must make all subsequent draws also.

$$\int_{z^{-1}(x_1)}^{x_1}\left[\int_{z(y)}^{\bar{x}}\left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^{M-1}\left\{\sum_{i=0}^{M-1}\binom{M-1}{i}\left(\frac{F(z(y))-F(y)}{F(x_T)-F(y)}\right)^{i}\left(\frac{F(x_T)-F(z(y))}{F(x_T)-F(y)}\right)^{M-1-i}\right.\right.$$
$$\left.\left.\times\left[F(z(y))^{T}+\left(1-F(z(y))^{T}\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]^{i}\right\}\left(\frac{F(z(y))^{T}-1}{F(z(y))-1}\right)\right]f(x)dx_T h_{mn}(y)dy$$

This is the probability of winning with a subsequent draw which is larger than z. Of course, this still requires that your first draw was less than z.

$$-\int_{0}^{x_1} B(y) f_{m:n}(y)dy \; - \int_{z^{-1}(x_1)}^{x_1} C\left(\frac{1-F^T(z(y))}{1-F(z(y))}\right) h_{m:n}(y)dy$$

Expected payment is partitioned into the expected bid cost (uniform price auction) and the expected research costs.

Expected Profit for any M and T is:

$$P \int_{z^{-1}(x_1)}^{x_1} \left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^{M-1} \left[F(x_1)\right]^{MT} h_{m:n}(y)dy$$

$$+P \int_{0}^{z^{-1}(x_1)} \left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^{M-1} \left\{ \sum_{i=0}^{M-1} \binom{M-1}{i} \left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)^{i} \left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)^{M-1-i} \right.$$
$$\left. \times \left[F(z(y))^T + \left(1-F(z(y))^T\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]^{i} \right\} h_{m:n}(y)dy$$

$$+P \int_{z^{-1}(x_1)}^{x_1} \left[\int_{x_1}^{z(y)} \left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^{M-1} \left[F(x_T)\right]^{T(M-1)} f_{1:T}(x)dx_T \right] h_{m:n}(y)dy$$

$$+P \int_{z^{-1}(x_1)}^{x_1} \left[\int_{z(y)}^{\bar{x}} \left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^{M-1} \left\{ \sum_{i=0}^{M-1} \binom{M-1}{i} \left(\frac{F(z(y))-F(y)}{F(x_T)-F(y)}\right)^{i} \left(\frac{F(x_T)-F(z(y))}{F(x_T)-F(y)}\right)^{M-1-i} \right.\right.$$
$$\left.\left. \times \left[F(z(y))^T + \left(1-F(z(y))^T\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]^{i} \right\} \left(\frac{F(z(y))^T-1}{F(z(y))-1}\right) f(x)dx_T \right] h_{m:n}(y)dy$$

$$-\int_{0}^{x_1} B(y) h_{m:n}(y)dy \; - \int_{z^{-1}(x_1)}^{x_1} C\left(\frac{1-F^T(z(y))}{1-F(z(y))}\right) h_{m:n}(y)dy$$

Demonstrating the bid inversion problem with larger M and T.

Below I have written the equations making up the expected profit of a firm when

M = 5, T = 5, P = 20, C = 1, and $X \sim U[0,1]$.

$$(1)\quad P\int_{z^{-1}(x_1)}^{x_1}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^{M-1}\left[F(x_1)\right]^{MT}h_{m:n}(y)dy$$

$$(2)\quad P\int_{0}^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^{M-1}\left\{\sum_{i=0}^{M-1}\binom{M-1}{i}\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)^{i}\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)^{M-1-i}\right.$$
$$\left.\times\left[F(z(y))^{T}+\left(1-F(z(y))^{T}\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]^{i}\right\}h_{m:n}(y)dy$$

$$(3)\quad P\int_{z^{-1}(x_1)}^{x_1}\left[\int_{x_1}^{z(y)}\left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^{M-1}\left[F(x_T)\right]^{T(M-1)}f_{1:T}(x)dx_T\right]h_{m:n}(y)dy$$

$$(4)\quad P\int_{z^{-1}(x_1)}^{x_1}\left[\int_{z(y)}^{\bar{x}}\left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^{M-1}\left\{\sum_{i=0}^{M-1}\binom{M-1}{i}\left(\frac{F(z(y))-F(y)}{F(x_T)-F(y)}\right)^{i}\left(\frac{F(x_T)-F(z(y))}{F(x_T)-F(y)}\right)^{M-1-i}\right.\right.$$
$$\left.\left.\times\left[F(z(y))^{T}+\left(1-F(z(y))^{T}\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]^{i}\right\}\left(\frac{F(z(y))^{T}-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

$$(5)\quad -\int_{0}^{x_1}B(y)h_{m:n}(y)dy-\int_{z^{-1}(x_1)}^{x_1}C\left(\frac{1-F^{T}(z(y))}{1-F(z(y))}\right)h_{m:n}(y)dy$$

First I will expand term #1 and substitute in for M = 5, T = 5 and P = 20.

$$(1)\qquad 20\int_{z^{-1}(x_1)}^{x_1}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^{4}\left[F(x_1)\right]^{25}h_{m:n}(y)dy$$

Expanding term #2 and substituting for M = 5 and T = 5 and P = 20:

(2a) $$20\int_0^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(z(y))}{1-F(y)}\right)^4 h_{m:n}(y)dy$$

(2b) $$20\int_0^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^4\left\{4\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)^3\right.$$
$$\left.\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]\right\}h_{m:n}(y)dy$$

(2c) $$20\int_0^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^4\left\{6\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)^2\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)^2\right.$$
$$\left.\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]^2\right\}h_{m:n}(y)dy$$

(2d) $$20\int_0^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^4\left\{4\left(\frac{F(z(y))-F(y)}{F(x_1)-F(y)}\right)^3\left(\frac{F(x_1)-F(z(y))}{F(x_1)-F(y)}\right)\right.$$
$$\left.\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]^3\right\}h_{m:n}(y)dy$$

(2e) $$20\int_0^{z^{-1}(x_1)}\left(\frac{F(z(y))-F(y)}{1-F(y)}\right)^4$$
$$\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]^4 h_{m:n}(y)dy$$

Expanding term (3) and substituting for M = 5 and T = 5 and P = 20.

(3) $$20\int_{z^{-1}(x_1)}^{x_1}\left[\int_{x_1}^{z(y)}\left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^4\left[F(x_T)\right]^{20} f_{1:T}(x)dx_T\right]h_{m:n}(y)dy$$

Expanding term (4) and substituting for M = 5 and T = 5 and P = 20.

(4a)
$$20\int_{z^{-1}(x_1)}^{x_1}\left[\int_{z(y)}^{\bar{x}}\left(\frac{F(x_T)-F(z(y))}{1-F(y)}\right)^4\left(\frac{F(z(y))^5-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

(4b)
$$20\int_{z^{-1}(x_1)}^{x_1}\left[\int_{z(y)}^{\bar{x}}\left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^4\left\{4\left(\frac{F(z(y))-F(y)}{F(x_T)-F(y)}\right)\left(\frac{F(x_T)-F(z(y))}{F(x_T)-F(y)}\right)^3\right.\right.$$
$$\left.\left.\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]\right\}\left(\frac{F(z(y))^5-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

(4c)
$$20\int_{z^{-1}(x_1)}^{x_1}\left[\int_{z(y)}^{\bar{x}}\left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^4\left\{6\left(\frac{F(z(y))-F(y)}{F(x_T)-F(y)}\right)^2\left(\frac{F(x_T)-F(z(y))}{F(x_T)-F(y)}\right)^2\right.\right.$$
$$\left.\left.\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]^2\right\}\left(\frac{F(z(y))^5-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

(4d)
$$20\int_{z^{-1}(x_1)}^{x_1}\left[\int_{z(y)}^{\bar{x}}\left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^4\left\{4\left(\frac{F(z(y))-F(y)}{F(x_T)-F(y)}\right)^3\left(\frac{F(x_T)-F(z(y))}{F(x_T)-F(y)}\right)\right.\right.$$
$$\left.\left.\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]^3\right\}\left(\frac{F(z(y))^5-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

(4e)
$$20\int_{z^{-1}(x_1)}^{x_1}\left[\int_{z(y)}^{\bar{x}}\left(\frac{F(z(y))-F(y)}{1-F(y)}\right)^4\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]^4\right.$$
$$\left.\times\left(\frac{F(z(y))^5-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

And finally the last term:

(5)
$$-\int_0^{x_1}B(y)f_{m:n}(y)dy-\int_{z^{-1}(x_1)}^{x_1}C\left(\frac{1-F^T(z(y))}{1-F(z(y))}\right)h_{m:n}(y)dy$$

Suppose the player bid as $\hat{x}$. Then after further factoring of terms, $\pi(\hat{x}, x_1) =$

$$(1) \qquad 20\int_{z^{-1}(x_1)}^{\hat{x}_1}\left(\frac{F(x_1)-F(y)}{1-F(y)}\right)^4\left[F(x_1)\right]^{25}h_{m:n}(y)dy$$

$$(2a) \qquad +\,20\int_{0}^{z^{-1}(x_1)}\left(\frac{F(x_1)-F(z(y))}{1-F(y)}\right)^4 h_{m:n}(y)dy$$

$$(2b) \qquad \begin{aligned}&+\,20\int_{0}^{z^{-1}(x_1)}4\left(\frac{F(z(y))-F(y)}{1-F(y)}\right)\left(\frac{F(x_1)-F(z(y))}{1-F(y)}\right)^3\\ &\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]h_{m:n}(y)dy\end{aligned}$$

$$(2c) \qquad \begin{aligned}&+\,20\int_{0}^{z^{-1}(x_1)}6\left(\frac{F(z(y))-F(y)}{1-F(y)}\right)^2\left(\frac{F(x_1)-F(z(y))}{1-F(y)}\right)^2\\ &\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]^2 h_{m:n}(y)dy\end{aligned}$$

$$(2d) \qquad \begin{aligned}&+\,20\int_{0}^{z^{-1}(x_1)}4\left(\frac{F(z(y))-F(y)}{1-F(y)}\right)^3\left(\frac{F(x_1)-F(z(y))}{1-F(y)}\right)\\ &\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]^3 h_{m:n}(y)dy\end{aligned}$$

$$(2e) \qquad \begin{aligned}&+\,20\int_{0}^{z^{-1}(x_1)}\left(\frac{F(z(y))-F(y)}{1-F(y)}\right)^4\\ &\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_1)-F(z(y))}{1-F(z(y))}\right)\right]^4 h_{m:n}(y)dy\end{aligned}$$

(3) $$+20\int_{z^{-1}(x_1)}^{\hat{x}_1}\left[\int_{x_1}^{z(y)}\left(\frac{F(x_T)-F(y)}{1-F(y)}\right)^4 [F(x_T)]^{20} f_{1:T}(x)dx_T\right]h_{m:n}(y)dy$$

(4a) $$+20\int_{z^{-1}(x_1)}^{\hat{x}_1}\left[\int_{z(y)}^{\bar{x}}\left(\frac{F(x_T)-F(z(y))}{1-F(y)}\right)^4\left(\frac{F(z(y))^5-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

(4b) $$+20\int_{z^{-1}(x_1)}^{\hat{x}_1}\left[\int_{z(y)}^{\bar{x}}4\left(\frac{F(z(y))-F(y)}{1-F(y)}\right)\left(\frac{F(x_T)-F(z(y))}{1-F(y)}\right)^3\right.$$

$$\left.\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]\left(\frac{F(z(y))^5-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

(4c) $$+20\int_{z^{-1}(x_1)}^{\hat{x}_1}\left[\int_{z(y)}^{\bar{x}}6\left(\frac{F(z(y))-F(y)}{1-F(y)}\right)^2\left(\frac{F(x_T)-F(z(y))}{1-F(y)}\right)^2\right.$$

$$\left.\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]^2\left(\frac{F(z(y))^5-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

(4d) $$+20\int_{z^{-1}(x_1)}^{\hat{x}_1}\left[\int_{z(y)}^{\bar{x}}4\left(\frac{F(z(y))-F(y)}{1-F(y)}\right)^3\left(\frac{F(x_T)-F(z(y))}{1-F(y)}\right)\right.$$

$$\left.\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]^3\left(\frac{F(z(y))^5-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

(4e) $$+20\int_{z^{-1}(x_1)}^{\hat{x}_1}\left[\int_{z(y)}^{\bar{x}}\left(\frac{F(z(y))-F(y)}{1-F(y)}\right)^4\right.$$

$$\left.\times\left[F(z(y))^5+\left(1-F(z(y))^5\right)\left(\frac{F(x_T)-F(z(y))}{1-F(z(y))}\right)\right]^4\left(\frac{F(z(y))^5-1}{F(z(y))-1}\right)f(x)dx_T\right]h_{m:n}(y)dy$$

(5) $$-\int_0^{\hat{x}_1}B(y)h_{m:n}(y)dy-\int_{z^{-1}(x_1)}^{\hat{x}_1}C\left(\frac{1-F^T(z(y))}{1-F(z(y))}\right)h_{m:n}(y)dy$$

Taking the derivative with respect to $\hat{x}$ we have $\pi_1(\hat{x},x_1) =$

$$(1)\qquad 20\left(\frac{F(x_1)-F(\hat{x})}{1-F(\hat{x})}\right)^4 \left[F(x_1)\right]^{25} h_{m:n}(\hat{x})$$

$$(3)\qquad +\,20\left[\int_{x_1}^{z(\hat{x})}\left(\frac{F(x_T)-F(\hat{x})}{1-F(\hat{x})}\right)^4 \left[F(x_T)\right]^{20} f_{1:T}(x)\,dx_T\right] h_{m:n}(\hat{x})$$

$$(4a)\qquad +\,20\left[\int_{z(\hat{x})}^{\bar{x}}\left(\frac{F(x_T)-F(z(\hat{x}))}{1-F(\hat{x})}\right)^4 \left(\frac{F(z(\hat{x}))^5-1}{F(z(\hat{x}))-1}\right) f(x)\,dx_T\right] h_{m:n}(\hat{x})$$

$$(4b)\qquad +\,20\left[\int_{z(\hat{x})}^{\bar{x}} 4\left(\frac{F(z(\hat{x}))-F(\hat{x})}{1-F(\hat{x})}\right)\left(\frac{F(x_T)-F(z(\hat{x}))}{1-F(\hat{x})}\right)^3 \right.$$
$$\left.\times\left[F(z(\hat{x}))^5+\left(1-F(z(\hat{x}))^5\right)\left(\frac{F(x_T)-F(z(\hat{x}))}{1-F(z(\hat{x}))}\right)\right]\left(\frac{F(z(\hat{x}))^5-1}{F(z(\hat{x}))-1}\right) f(x)\,dx_T\right] h_{m:n}(\hat{x})$$

$$(4c)\qquad +\,20\left[\int_{z(\hat{x})}^{\bar{x}} 6\left(\frac{F(z(\hat{x}))-F(\hat{x})}{1-F(\hat{x})}\right)^2\left(\frac{F(x_T)-F(z(\hat{x}))}{1-F(\hat{x})}\right)^2 \right.$$
$$\left.\times\left[F(z(\hat{x}))^5+\left(1-F(z(\hat{x}))^5\right)\left(\frac{F(x_T)-F(z(\hat{x}))}{1-F(z(\hat{x}))}\right)\right]^2\left(\frac{F(z(\hat{x}))^5-1}{F(z(\hat{x}))-1}\right) f(x)\,dx_T\right] h_{m:n}(\hat{x})$$

$$(4d)\qquad +\,20\left[\int_{z(\hat{x})}^{\bar{x}} 4\left(\frac{F(z(\hat{x}))-F(\hat{x})}{1-F(\hat{x})}\right)^3\left(\frac{F(x_T)-F(z(\hat{x}))}{1-F(\hat{x})}\right) \right.$$
$$\left.\times\left[F(z(\hat{x}))^5+\left(1-F(z(\hat{x}))^5\right)\left(\frac{F(x_T)-F(z(\hat{x}))}{1-F(z(\hat{x}))}\right)\right]^3\left(\frac{F(z(\hat{x}))^5-1}{F(z(\hat{x}))-1}\right) f(x)\,dx_T\right] h_{m:n}(\hat{x})$$

$$(4e)\qquad +\,20\left[\int_{z(\hat{x})}^{\bar{x}} \left(\frac{F(z(\hat{x}))-F(\hat{x})}{1-F(\hat{x})}\right)^4 \right.$$
$$\left.\times\left[F(z(\hat{x}))^5+\left(1-F(z(\hat{x}))^5\right)\left(\frac{F(x_T)-F(z(\hat{x}))}{1-F(z(\hat{x}))}\right)\right]^4\left(\frac{F(z(\hat{x}))^5-1}{F(z(\hat{x}))-1}\right) f(x)\,dx_T\right] h_{m:n}(\hat{x})$$

$$(5)\qquad -\,B(\hat{x})h_{m:n}(\hat{x}) \;-\; C\left(\frac{1-F^T(z(\hat{x}))}{1-F(z(\hat{x}))}\right) h_{m:n}(\hat{x})$$

For a pure-strategy equilibrium, we require $\pi_1(\hat{x}, x_1) = 0$ when evaluated at $\hat{x} = x_1$.

$$B(x_1) = 20\left[\int_{x_1}^{z(x_1)} \left(\frac{F(x_T) - F(x_1)}{1 - F(x_1)}\right)^4 [F(x_T)]^{20} f_{1:T}(x) dx_T\right]$$

$$+ 20\left[\int_{z(x_1)}^{\bar{x}} \left(\frac{F(x_T) - F(z(x_1))}{1 - F(x_1)}\right)^4 \left(\frac{F(z(x_1))^5 - 1}{F(z(x_1)) - 1}\right) f(x) dx_T\right]$$

$$+ 20\left[\int_{z(x_1)}^{\bar{x}} 4\left(\frac{F(z(x_1)) - F(x_1)}{1 - F(x_1)}\right)\left(\frac{F(x_T) - F(z(x_1))}{1 - F(x_1)}\right)^3\right.$$

$$\left.\times\left[F(z(x_1))^5 + \left(1 - F(z(x_1))^5\right)\left(\frac{F(x_T) - F(z(x_1))}{1 - F(z(x_1))}\right)\right]\left(\frac{F(z(x_1))^5 - 1}{F(z(x_1)) - 1}\right) f(x) dx_T\right]$$

$$+ 20\left[\int_{z(x_1)}^{\bar{x}} 6\left(\frac{F(z(x_1)) - F(x_1)}{1 - F(x_1)}\right)^2 \left(\frac{F(x_T) - F(z(x_1))}{1 - F(x_1)}\right)^2\right.$$

$$\left.\times\left[F(z(x_1))^5 + \left(1 - F(z(x_1))^5\right)\left(\frac{F(x_T) - F(z(x_1))}{1 - F(z(x_1))}\right)\right]^2 \left(\frac{F(z(x_1))^5 - 1}{F(z(x_1)) - 1}\right) f(x) dx_T\right]$$

$$+ 20\left[\int_{z(x_1)}^{\bar{x}} 4\left(\frac{F(z(x_1)) - F(x_1)}{1 - F(x_1)}\right)^3 \left(\frac{F(x_T) - F(z(x_1))}{1 - F(x_1)}\right)\right.$$

$$\left.\times\left[F(z(x_1))^5 + \left(1 - F(z(x_1))^5\right)\left(\frac{F(x_T) - F(z(x_1))}{1 - F(z(x_1))}\right)\right]^3 \left(\frac{F(z(x_1))^5 - 1}{F(z(x_1)) - 1}\right) f(x) dx_T\right]$$

$$+ 20\left[\int_{z(x_1)}^{\bar{x}} \left(\frac{F(z(x_1)) - F(x_1)}{1 - F(x_1)}\right)^4\right.$$

$$\left.\times\left[F(z(x_1))^5 + \left(1 - F(z(x_1))^5\right)\left(\frac{F(x_T) - F(z(x_1))}{1 - F(z(x_1))}\right)\right]^4 \left(\frac{F(z(x_1))^5 - 1}{F(z(x_1)) - 1}\right) f(x) dx_T\right]$$

$$- \left(\frac{1 - F^T(z(x_1))}{1 - F(z(x_1))}\right)$$

Because we are assuming that x is distributed uniform[0,1], we can further simplify:

$$B(x_1) = 100\left[\int_{x_1}^{z(x_1)} \left(\frac{x_T - x_1}{1-x_1}\right)^4 [x_T]^{24}\, dx_T\right]$$

$$+ 20\left[\int_{z(x_1)}^{\bar{x}} \left(\frac{x_T - z(x_1)}{1-x_1}\right)^4 \left(\frac{z(x_1)^5 - 1}{z(x_1)-1}\right) dx_T\right]$$

$$+ 80\left[\int_{z(x_1)}^{\bar{x}} \left(\frac{z(x_1) - x_1}{1-x_1}\right)\left(\frac{x_T - z(x_1)}{1-x_1}\right)^3\right.$$

$$\left.\times\left[z(x_1)^5 + \left(1 - z(x_1)^5\right)\left(\frac{x_T - z(x_1)}{1-z(x_1)}\right)\right]\left(\frac{z(x_1)^5 - 1}{z(x_1)-1}\right) dx_T\right]$$

$$+ 120\left[\int_{z(x_1)}^{\bar{x}} \left(\frac{z(x_1) - x_1}{1-x_1}\right)^2\left(\frac{x_T - z(x_1)}{1-x_1}\right)^2\right.$$

$$\left.\times\left[z(x_1)^5 + \left(1 - z(x_1)^5\right)\left(\frac{x_T - z(x_1)}{1-z(x_1)}\right)\right]^2\left(\frac{z(x_1)^5 - 1}{z(x_1)-1}\right) dx_T\right]$$

$$+ 80\left[\int_{z(x_1)}^{\bar{x}} \left(\frac{z(x_1) - x_1}{1-x_1}\right)^3\left(\frac{x_T - z(x_1)}{1-x_1}\right)\right.$$

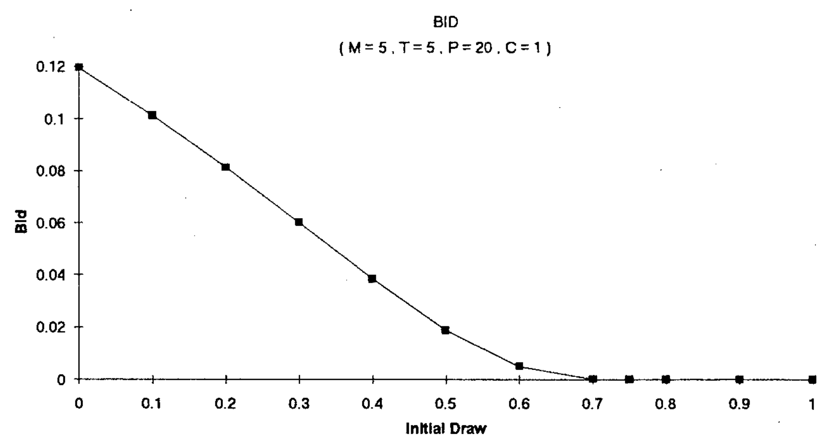
$$\left.\times\left[z(x_1)^5 + \left(1 - z(x_1)^5\right)\left(\frac{x_T - z(x_1)}{1-z(x_1)}\right)\right]^3\left(\frac{z(x_1)^5 - 1}{z(x_1)-1}\right) dx_T\right]$$

$$+ 20\left[\int_{z(x_1)}^{\bar{x}} \left(\frac{z(x_1) - x_1}{1-x_1}\right)^4\left[z(x_1)^5 + \left(1 - z(x_1)^5\right)\left(\frac{x_T - z(x_1)}{1-z(x_1)}\right)\right]^4\left(\frac{z(x_1)^5 - 1}{z(x_1)-1}\right) dx_T\right]$$

$$- \left(\frac{1 - z(x_1)^5}{1 - z(x_1)}\right)$$

Solution
(M = 5 , T = 5 , P = 20 , C = 1)

X1	Z(X1)	BID
0	0.820324	0.11944556
0.1	0.817934	0.1012175
0.2	0.814821	0.08127739
0.3	0.810635	0.05999164
0.4	0.804796	0.03840014
0.5	0.796324	0.018783
0.6	0.78356	0.004988
0.7	0.763815	0.000143
0.75	0.75	0
0.8	0.8	0
0.9	0.9	0
1	1	0

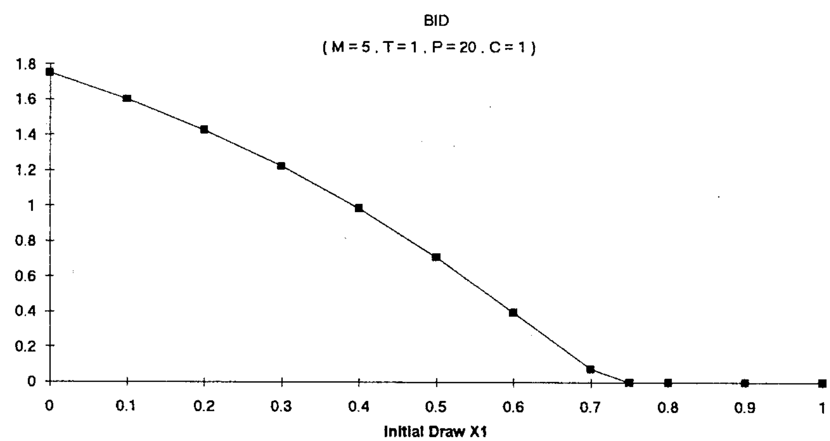


This solution obviously illustrates the bid inversion problem and clearly cannot be an equilibrium bidding function.

For comparison, below I have plotted the solution for the case when

$$T = 1\ ,\ M = 5\ ,\ P = 20\ ,\ \text{and}\ C = 1.$$

X1	Z(X1)	BID
0	0.840061	1.751168
0.1	0.841956	1.600946
0.2	0.843981	1.426636
0.3	0.846068	1.223254
0.4	0.848023	0.98544
0.5	0.849264	0.708845
0.6	0.847691	0.395073
0.7	0.829538	0.07601
0.75	0.75	0
0.8	0.8	0
0.9	0.9	0
1	1	0



Again, we have the bid inversion problem, demonstrating that this cannot be an equilibrium even when firms are allowed to conduct research following entry.

Appendix 1C

Entry Auctions

in

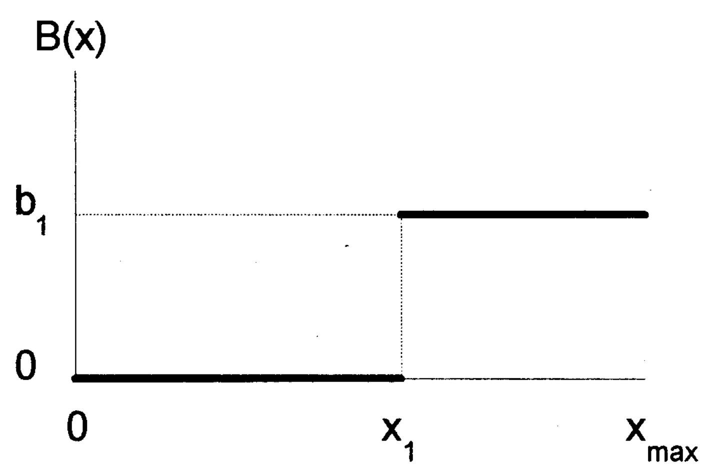
Tournaments with Discrete Bids

In chapter two I proved when potential contestants have different technology endowments there is no symmetric bidding equilibrium. However, this result applies only when the support of possible bids is continuous. If we restrict each contestant's entry bid to discrete monetary units then there is always a *weakly* increasing pure-strategy bidding equilibrium for the entry auction. Unfortunately, this only slightly improves the outcome of the entry auction because a necessary component of the pure-strategy equilibrium is that there must be significant probabilities of tie bids among different firms. In other words, the auction is still not efficient because there is no guarantee that the best M firms will be selected due to tie bids. Additionally, while reducing the bidding increment will reduce the probability of ties, the evidence strongly suggests that reducing the bidding increment also lowers the entry bids of the best firms. Thus, there is a trade-off between improving the sponsor's probability of getting the best entrants versus collecting larger entry fees from the top bidding firms.

This chapter briefly delves into the nature of discrete pure-strategy bidding equilibria. If the support set of possible innovations is finite, I prove that as the bidding increments become finer the maximum bid of any firm necessarily converges to zero. On the other hand, if the support set of innovations is continuous or infinite, then I will offer evidence suggesting that once again the maximum bid converges to zero as the bidding increment goes to zero. Unfortunately, due to the

complexity of the simultaneous equations, I am not able to offer a complete proof of convergence in the continuous case, though all available evidence seems to suggest that this does happen. Thus, technological starting differences continue to be a problem for sponsors even when bidding is discrete. The sponsor apparently faces a substantial trade-off between getting the best firms and collecting larger entry fees.

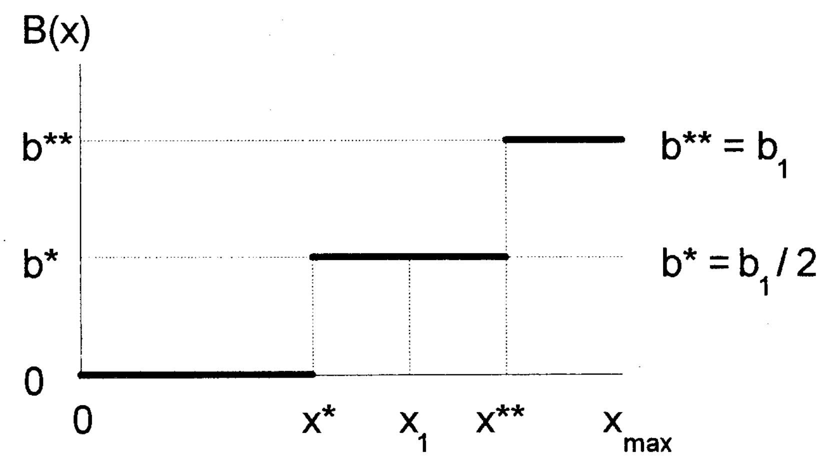
We can restore a pure-strategy equilibrium by allowing firms to raise and lower their bids only by discrete amounts. In all of the analyses presented in this chapter, I will be assuming that research is not done following entry so we can address the "pure" adverse selection model. Suppose we allow only two bids: 0 and $b_1 < P$, which is essentially like charging a single entry fee. However, we will assume that if there are more than M firms that bid b_1 then M of those firms will be selected at random. With only the options of bidding 0 or b_1, then the pure-strategy equilibrium for each firm will appear as we have depicted below.



In the figure above, I have drawn a pure-strategy bidding equilibrium that is *weakly* increasing in innovations: $(\forall\, x, x^* \in X), x^* > x \Rightarrow B(x^*) \geq B(x)$.

For any pure strategy equilibrium, any firm with $x = 0$ always bids 0 because this firm cannot win the prize since the probability of a tie is zero with a continuous distribution. As x gets larger, eventually there is a "critical" value x_1 where the risk of tying with a bid of 0 and not being picked for entry when x_1 is the best firm exactly equals the risks of bidding b_1 and guaranteeing entry when x_1 is the largest but paying an entry fee on occasions when it is not the largest but gains entry anyway. Now all values of x to the left of x_1 bid zero and all values to the right would bid b_1. This is a pure-strategy equilibrium because *ex ante* every firm earns strictly less expected profit if it deviates from the equilibrium. Firms that bid zero would earn less by bidding a positive amount because they gain entry too often in comparison to their frequency of being the largest innovation. Firms that bid strictly positive amounts would earn less by bidding zero because they would fail to gain entry enough times when they are actually the best innovation to make it worthwhile to bid a positive amount.

Suppose now we add another possible bid, b^*, halfway between 0 and b_1 :



When we allow a bid that is less b_1, then obviously x_1 prefers this new lower bid, but so do other values of x on either side of our original critical value of x_1. Thus, by reducing the increments between bids we make it less expensive to raise your bid so we shorten the intervals between critical values of x because now it takes a smaller probability of a tie at any single bid to induce a firm to raise its bid to the next higher bid. Therefore, as the bidding increment decreases, the "partitions" of x that bid any particular bid will decrease and the probability of ties at any particular bid also decreases. We are interested in showing exactly what happens as the increments between acceptable bids shrinks.

A STYLIZED EXAMPLE

Consider the following stylized and simplified game. Suppose that innovations are drawn *without replacement* from an "bucket" containing exactly four

balls: $x \in \{0,1,2,3\}$. Let us assume there are three firms and each firm selects a single ball. Since the draws are made without replacement, there is no possibility of a tie and there is exactly 4 possible combinations of draws by the three firms: (0,1,2) , (0,1,3) , (0,2,3) , and (1,2,3). The sponsor selects exactly two firms (M = 2) as finalists. The sponsor selects entrants according to who "bids" the highest amount to gain entry. If two firms bid higher than a third firm, then those two high-bidding firms gain entry. If there is a tie in the bidding, then the tie is broken randomly by the sponsor. Any firm that is not selected as a finalist does not have to pay its entry bid, but *both* of the finalists must pay their respective bids. Once the two finalists have been selected and paid their bids, the sponsor evaluates each of their draws and awards \$100 to the firm with the largest draw (P = 100). Note that this stylized example game is analogous to our procurement contest except that in this simplified example we have a finite population of possible innovations (four). Now suppose that the tournament sponsor only allowed bids in \$25 increments. Can we find a pure-strategy equilibrium for the bidding? Yes, the following is a pure-strategy equilibrium:

$$B(0) = 0 \ , \ B(1) = 0 \ , \ B(2) = 0 \ , \ B(3) = \$25$$

First, any firm that draws 0 cannot win the prize so $B(0) = 0$. Since a firm with a draw of 3 always wins given entry, if it bids 0 it has only a 67% chance of being randomly selected for entry so it has an expected profit of \$67. On the other hand,

when firm 3 bids \$25 it always gains entry with an expected profit of \$75, so it prefers to bid \$25. A firm with a draw of 2 can only win when nobody else draws a 3, but this only happens 33% of the time from the firm's perspective. Therefore, if it bid \$25 it would always gain entry, but lose 67% of the time for an expected profit of: $(100 - 25)(.33) - 25(.67) = \$8.35$. However, if firm 2 bids 0 it gains entry (and wins) 67% of the time when no firm draws a 3, and it cannot win when another firm draws a 3, regardless of whether or not it gains entry. Therefore, its expected profit from a bid of 0 is: $(100)(.33)(.67) = \$22.11$, so it prefers to bid 0. Obviously, if a firm with a 2 prefers to bid zero, then the firm with a 1 also prefers to bid zero. Therefore, we have established that (0,0,0,\$25) is a pure-strategy bidding equilibrium.

Let us tweak our stylized example just a bit and say that any bids are allowed as long as they are made in \$1 increments. How does our equilibrium change? The following is a pure-strategy equilibrium for this game:

$$B(0) = 0 \ , \ B(1) = 0 \ , \ B(2) = \$1 \ , \ B(3) = \$1$$

Again we know that any firm with a draw of 0 can never win, so $B(0) = 0$. Now let us look at the game from the perspective of a firm holding a draw of 1. When can it win? Only if all firms bid 0 and it is randomly selected along with the firm with 0. However, since two of the draws bid positive amounts, then all firms never bid zero which implies that a firm with a draw of 1 will never win. Therefore it prefers to bid

zero to avoid ever paying an entry fee, *B(1)* = \$0. For the firm with the highest draw of 3, it was willing to bid \$25 in this game so it clearly is willing to bid \$1. Moreover, since only two firms are bidding positive amounts, entry is always ensured with a \$1 bid, so *B(3)* = \$1. Finally, for the firm that draws a 2, just like the previous example a bid of zero would have an expected value of \$22.11. But a bid of \$1 ensures entry so its expected value is: (100-1)(.33) -1(.67) = \$32 which implies that *B(2)* = \$1. Therefore, we again have a pure-strategy equilibrium.

However two important things have changed in our examples from the sponsor's perspective when he reduced the increment between acceptable bids from \$25 to \$1. First, the highest bid fell from \$25 -- which is bad from the sponsor's point of view. Second, now there are *two* firms that bid positive amounts so that the sponsor in the second example always ensures that he has at least one of the top two firms as a finalist. In contrast, in the first game if no firm drew a 3, then there was a 33% chance that a firm drew a 2 but was not selected as a finalist. Finally, it is easy to see at this point that no matter how much smaller we make the bidding increments in this example, any firm drawing a 0 or 1 will *always* bid zero and firm drawing one of the top two innovations will always bid the smallest positive increment. So, if bidding increments are allowed in pennies the equilibrium is:

B(0) = 0 , *B(1)* = 0 , *B(2)* = 1 penny , *B(3)* = 1 penny

As the bidding units converge to zero, this pattern is preserved and all bids converge to zero.

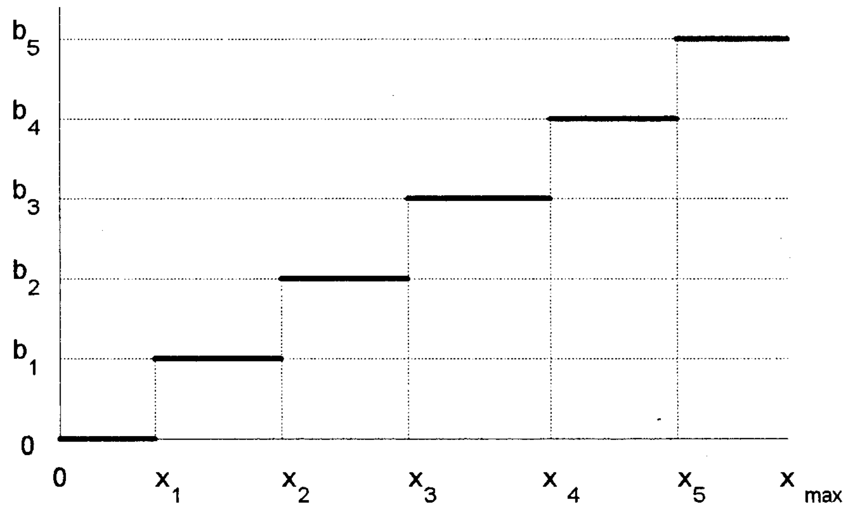
This intuition extends to any finite set of innovations. As long as there are only a finite set of possible innovations, then they will occupy a finite number of possible bids. In any situation where draws are made without replacement (i.e., no "tie" innovations) then a firm with the smallest possible draw will always bid zero since it can never win the tournament. Additionally, there will never be a "gap" in the bidding because then any firm bidding at the top of the gap could strictly increase its expected profits by lowering its bid into the gap. Therefore, there is always at least one innovation occupying each bidding increment starting at zero and bidding upward until we exhaust the finite set of innovations. But since the innovation set is finite, then the bids must all converge to zero as the bidding increment converges to zero. As a short proof, suppose not. Suppose the largest bid converged to some strictly positive number, $V > 0$. Now suppose that there are W possible innovations in the finite set. Then if the bidding increment ever falls below V/W, this would require the highest bid to be less than V because there are no gaps in the bidding and the bidding starts at zero. So there always exists some bidding increment small enough to drive the bids below any given positive number. Therefore, all bids must converge to zero as the bidding increment converges to

zero if firms are assumed to be drawing innovations without replacement from a finite set of innovations.

DISCRETE BIDS WITH CONTINUOUS INNOVATIONS

Now suppose again that the innovation set is continuous, but the bidding units are discrete. Just like in the previous example, I would like to show as the increment between bids is reduced this results in a decline in the highest bid. In fact, I would like to show that as the bidding increments become infinitesimally small all bids converge to zero. Unfortunately, due to the complexity of the equations I was never able to prove this result outright, however below I present strong evidence to suggest that this is the case.

For the analysis, I will label the acceptable bids in order of increasing size with subscripts: $0 < b_1 < b_2 < \ldots < b_I$ where b_I is the highest bid made by any firm. Also, to limit notation I will assume there are $(N + 1)$ firms so that each firm has N other firms it is competing against for entry. I can restrict the support of bids to $b < P$ since any firm bidding more than P guarantees itself a loss if it gains entry. Finally, in all of the following we will be defining the "critical" values of x, which are also ordered with subscripts, by their indifference between bids. Specifically, we will say that x_i is indifferent between bidding b_i and b_{i-1}. This is depicted in the figure below:



For all critical values of x_i in any equilibrium (i.e., x is just indifferent between bidding its "high" bid of b_i and risking a tie with its "low" bid of b_{i-1}), x_i only wins the tournament if it is the *largest* of all (N+1) values of x. This is because when x_i bids b_i it is the *smallest* x that is bidding b_i , so any other firm bidding $\geq b_i$ must be larger than x_i. So in order for x_i to win all other firms must bid $< b_i$ which means that x_i is the largest of all (N+1) firms. On the other hand, if x_i bids its "low" bid of b_{i-1} then any firm larger than x_i will bid strictly greater than b_{i-1} so again x_i cannot win unless it is the largest x. In contrast, being the largest of all innovations is not a requirement to win for *non-critical* values of x because for any x that is interior to a bidding interval it is possible that it is not the largest x's but still wins the prize because it was one of *more than M* firms which tied for the highest bid, and there

was at least one other firm which bid the same amount and was larger than x_i but did not get picked at random for entry.

In equilibrium since the critical value of x_i is exactly indifferent between bidding b_i and bidding b_{i-1}. All firms with smaller x values strictly prefer to bid the lower bid, b_{i-1} over b_i , while all firms with larger x values strictly prefer to bid the higher bid, b_i over the lower bid b_{i-1}.

Again, I define the following terms for the analysis:

$B(x_i)$:= the pure-strategy equilibrium bidding function. In order to shorten notation, we will generally refer to bids using the lower case b.

$\theta(b)$:= the unconditional probability of gaining entry with a bid of b.

$\Omega(b_{i-1}| x_i)$: = the conditional probability of gaining entry with a bid of b_{i-1} , *given* that x_i is the largest of all innovations.

Some innovation x_i is a critical value for bid i if and only if the following inequality holds.

$$P\left[F(x_i)\right]^N - b_i\theta(b_i) \;=\; P\left[F(x_i)\right]^N \Omega(b_{i-1}|x_i) - b_{i-1}\theta(b_{i-1})$$

This equation looks a little odd, so I will explain it. First, it just says x_i's expected profits from a bid of b_i are equal to its expected profits from a bid of b_{i-1}. On the left-hand side, x_i bidding b_i , can only win if it is larger than all other draws and b_i was the singularly largest bid submitted. Therefore, with a bid of b_i, critical value x_i gains entry and wins the prize only when it is larger than all other firms -- and that

probability is $[F(x_i)]^N$. With a bid of b_i the firm must pay the entry fee with probability $\theta(b_i)$. On the right side, the first term must be multiplied by $\Omega(b_{i-1}| x_i)$ because now if you are the largest firm you still may not gain entry due to the probability of more than M firms tying with bids at b_{i-1}.

The formulas for $\theta(b_i)$, the unconditional probability of gaining entry are:

For $i = I$

$$\theta(b_I) = \sum_{k=0}^{M-1} \binom{N}{k} [1 - F(x_I)]^k [F(x_I)]^{N-k} + \sum_{k=M}^{N} \left(\frac{M}{k+1}\right) \binom{N}{k} [1 - F(x_I)]^k [F(x_I)]^{N-k}$$

For $0 < i < I$:

$$\theta(b_i) = \sum_{j=0}^{M-1} \left[\binom{N}{j} (1 - F(x_{i+1}))^j (F(x_{i+1}))^{N-j} \right.$$
$$\times \left\{ \sum_{k=M-j}^{N-j} \binom{N-j}{k} \left(\frac{F(x_{i+1}) - F(x_i)}{F(x_{i+1})} \right)^k \left(\frac{F(x_i)}{F(x_{i+1})} \right)^{N-j-k} \left(\frac{M-j}{k+1} \right) \right.$$
$$\left.\left. + \sum_{k=0}^{M-j-1} \binom{N-j}{k} \left(\frac{F(x_{i+1}) - F(x_i)}{F(x_{i+1})} \right)^k \left(\frac{F(x_i)}{F(x_{i+1})} \right)^{N-j-k} \right\} \right]$$

For $i = 0$ (i.e., $b = 0$):
$$\theta(b_i) = \sum_{j=0}^{M-1} \left[\binom{N}{j} (1 - F(x_1))^j (F(x_1))^{N-j} \left(\frac{M-j}{N-j+1} \right) \right]$$

The formulas for $\Omega(b_i| x_{i-1})$ are:

For $1 < i < I$:
$$\Omega(b_{i-1}|x_i) = \sum_{k=M}^{N} \binom{N}{k} \left(\frac{F(x_i) - F(x_{i-1})}{F(x_i)} \right)^k \left(\frac{F(x_{i-1})}{F(x_i)} \right)^{N-k} \left(\frac{M}{k+1} \right)$$
$$+ \sum_{k=0}^{M-1} \binom{N}{k} \left(\frac{F(x_i) - F(x_{i-1})}{F(x_i)} \right)^k \left(\frac{F(x_{i-1})}{F(x_i)} \right)^{N-k}$$

For $i = 1$: $\Omega(0|x_1) = \left(\frac{M}{N+1}\right)$

Note that if the Ith critical value x_I is $\bar{x}$, this implies that $\bar{x}$ is the only possible firm that submits a bid of b_I therefore $\theta(b_I) = 1$. When this situation occurs, this also implies that: $\Omega(b_{I-1}|x_I = \bar{x}) = \theta(b_{I-1})$. Therefore when $x_I = \bar{x}$, the equation defining the Ith critical value reduces to just:

$$P - b_I = [P - b_{I-1}]\theta(b_{I-1})$$

I will label the maximum incremental bid I as *"J"* when $x_I = \bar{x}$ and I will only talk about *"J"* when the largest critical value of x singularly occupies the highest bid in the equilibrium. Therefore, when $J = 1$, all values of x bid 0, and only $\bar{x}$ singularly bids b_1. Similarly when $J = 2$ then all values of x bid either 0 or b_1, while only a firm holding the largest possible innovation $\bar{x}$ bids b_2.

Using our equilibrium equation for the first critical value of x, we can explicitly solve for the maximum incremental bid when $J = 1$ as follows:

$PF(\bar{x})^N - b_1\theta(b_1) = PF(\bar{x})^N \Omega(0|\bar{x})$ rearranging,

$PF(\bar{x})^N [1 - \Omega(0|\bar{x})] = b_1\theta(b_1)$, but $F(\bar{x})^N = 1$ and $\theta(b_1) = 1$

Thus, $P\left[1 - \left(\frac{M}{N+1}\right)\right] = b_{I=1}^{\max}$

As an example, suppose the sponsor was offering a \$30 prize and three firms wanted to enter the tournament but the sponsor had to restrict entry to only two firms. In this case, if the sponsor allowed bids to be only in increments of \$10, then the

bidding equilibrium would have all firms bid zero except a firm that drew the largest possible value of $\bar{x}$ and that firm would be indifferent between bidding zero and \$10.

Suppose we reduced our bidding increment slightly below \$10, then some smaller values of x less than $\bar{x}$ would also prefer to bid b_1. However, eventually as we continued to reduce the bidding increment then at some point the bidding increment would be small enough and there would be so many others values of x that were bidding b_1 that a firm with $\bar{x}$ would become indifferent between bidding b_1 and raising its bid to $2b_1 = b_{J=2}^{\max}$ and we would label this new bidding equilibrium by $J = 2$. The question we would like to know is whether $b_{J=2}^{\max}$ *or* $b_{J=1}^{\max}$ is larger. The following proof demonstrates that the maximum bid must always *decrease* as J goes from 1 to 2.

Lemma C1: *In order to induce $J = 2$ in the bidding the sponsor must drop the bidding increment to less than half of what is required for $J = 1$. Therefore, the maximum possible bid always falls from $J = 1$ to $J = 2$.*

Proof: When $J = 1$, the largest bid is: $b_{J=1}^{\max} = P\left[1-\left(\frac{M}{N+1}\right)\right]$

When $J=1$ the expected profit from the largest $\bar{x}$ is just: $\pi(1) = P - \bar{b}_1 = P\left[\frac{M}{N+1}\right]$

When $J = 2$, the equations defining the critical values of x are:

$$P[F(x_2)]^N - b_2\theta(b_2) = P[F(x_2)]^N\,\Omega(b_1|x_2) - b_1\theta(b_1)$$

$$P[F(x_1)]^N - b_1\theta(b_1) = P[F(x_1)]^N\,\Omega(0|x_1)$$

From the bottom equation we know that: $PF_1^N\left[\frac{N+1-M}{N+1}\right]=\frac{\overline{b}_2}{2}\theta(b_1)$

From the right-hand side of the top equation when J = 2 we know that expected profit for the largest $\overline{x}$ is just: $P\theta(b_1)-\frac{\overline{b}_2}{2}\theta(b_1)$

So we know the expected profit for J = 2 is: $\pi(2)=P\theta(b_1)-PF_1^N\left[\frac{N+1-M}{N+1}\right]$

In both cases of $J=1$ and $J=2$, the top value of $\overline{x}$ always gains entry, since it singularly occupies the highest possible bid. The J with the largest expected profit is associated with the smallest bid since the profit of $\overline{x}$ is always $P-\overline{b}$. So we will compare the two expected profits to see which is largest.

$$\pi(2)=P\theta(b_1)-PF_1^N\left[\frac{N+1-M}{N+1}\right]\ \begin{matrix}>\\<\end{matrix}\ P\left[\frac{M}{N+1}\right]=\pi(1)$$

Canceling out "P" to compare: $\theta(b_1)-F_1^N\left[\frac{N+1-M}{N+1}\right]\ \begin{matrix}>\\<\end{matrix}\ \left[\frac{M}{N+1}\right]$

Rearranging terms: $$\theta(b_1)-F_1^N\left[\frac{N+1-M}{N+1}\right]-\left[\frac{M}{N+1}\right]\ \begin{matrix}>\\<\end{matrix}\ 0$$

But the formula for $\theta(b_1)$ is:

$$\theta(b_1)=\sum_{K=0}^{M-1}\binom{N}{K}(1-F_1)^K(F_1)^{N-K}+\sum_{K=M}^{N}\binom{N}{K}(1-F_1)^K(F_1)^{N-K}\left(\frac{M}{K+1}\right)$$

Since $1=\left[\frac{N+1-M}{N+1}\right]+\left[\frac{M}{N+1}\right]$ we can substitute this into our first half of the formula to get:

$$\theta(b_1)=\left[\frac{N+1-M}{N+1}\right]\sum_{K=0}^{M-1}\binom{N}{K}(1-F_1)^K(F_1)^{N-K}+\left[\frac{M}{N+1}\right]\sum_{K=0}^{M-1}\binom{N}{K}(1-F_1)^K(F_1)^{N-K}$$

$$+\sum_{K=M}^{N}\binom{N}{K}(1-F_1)^K(F_1)^{N-K}\left(\frac{M}{K+1}\right)$$

However, $\frac{M}{K+1}\geq\frac{M}{N+1}$ so we can substitute into the far right term and combine:

$$\theta(b_1)>\left[\frac{N+1-M}{N+1}\right]\sum_{K=0}^{M-1}\binom{N}{K}(1-F_1)^K(F_1)^{N-K}+\left[\frac{M}{N+1}\right]\sum_{K=0}^{N}\binom{N}{K}(1-F_1)^K(F_1)^{N-K}$$

From the Binomial Theorem the right-hand term reduces to just: $\frac{M}{N+1}$

Therefore: $\theta(b_1)>\left[\frac{N+1-M}{N+1}\right]\sum_{K=0}^{M-1}\binom{N}{K}(1-F_1)^K(F_1)^{N-K}+\left[\frac{M}{N+1}\right]$

Now looking only at the left term when K = 0 it is: $F_1{}^N\left[\frac{N+1-M}{N+1}\right]$

Hence: $\theta(b_1)>\left[\frac{N+1-M}{N+1}\right]\sum_{K=1}^{M-1}\binom{N}{K}(1-F_1)^K(F_1)^{N-K}+F_1{}^N\left[\frac{N+1-M}{N+1}\right]+\left[\frac{M}{N+1}\right]$

Which implies:

$$\theta(b_1)-F_1{}^N\left[\frac{N+1-M}{N+1}\right]-\left[\frac{M}{N+1}\right]>\left[\frac{N+1-M}{N+1}\right]\sum_{K=1}^{M-1}\binom{N}{K}(1-F_1)^K(F_1)^{N-K}>0$$

Which implies: $\pi(2)>\pi(1)$ which of course means that $\bar{b}_1>\bar{b}_2$ **Q.E.D.**

To illustrate the previous proof, suppose that the sponsor offers a prize of \$100, there are three firms that want to enter the contest, and the sponsor only allows two to enter. If the sponsor sets the bidding increment at \$100/3 then all firms will bid zero, unless some firm has drawn the maximum possible innovation value -- in which case it will be indifferent between a bid of zero and a bid of

\$33.33. Now, if the sponsor were to drop the bidding increment down to \$8.35 then we would have a $J = 2$ scenario where all firms with innovations such that $F(x) < .47715$ would bid zero and all other firms with better innovations would bid \$8.35. Only the firm with the highest possible innovation draw would be indifferent between a bid of \$8.35 and a bid of \$16.70. However, the maximum possible bid *fell* from \$33.33 all the way down to \$16.70 between $J = 1$ and $J = 2$, which we proved would happen in the previous Lemma. If we picked a bidding increment between \$33.33 and \$8.35 then we would have split the critical values above. For example, if the sponsor had chosen an increment of \$20, then all firms with innovations less than $F(x) < .767592$ would bid zero while all larger firms would bid \$20. (Note in this case that .767592 is the "critical" value defined by the equilibrium equation and $\bar{x}$ is not a critical value because it strictly prefers a bid of \$20 above all other bids.)

We have proven that as the sponsor initially reduces the bidding increment the maximum bids begin to fall. We would like to know if this is a general phenomenon suggesting that there is a trade-off between smaller bidding intervals and bid revenues. In other words, does this tendency of the largest bid to fall carry over when J equals some arbitrary value of I? Assuming there are I positive bids which are made in equilibrium, we will have I equilibrium equations (one for each critical value) which we can stack as shown.

$$P[F(x_I)]^N - b_I\theta(b_I) = P[F(x_I)]^N \Omega(b_{I-1}|x_I) - b_{I-1}\theta(b_{I-1})$$

$$P[F(x_{I-1})]^N - b_{I-1}\theta(b_{I-1}) = P[F(x_{I-1})]^N \Omega(b_{I-2}|x_{I-1}) - b_{I-2}\theta(b_{I-2})$$

$$\bullet \qquad \bullet$$

$$\bullet \qquad \bullet$$

$$P[F(x_2)]^N - b_2\theta(b_2) = P[F(x_2)]^N \Omega(b_1|x_2) - b_1\theta(b_1)$$

$$P[F(x_1)]^N - b_1\theta(b_1) = P[F(x_1)]^N \Omega(0|x_1)$$

Summing up all of these equilibrium conditions and canceling like terms:

$$P\sum_{i=1}^{I}[F(x_i)]^N - b_I\theta(b_I) = P\sum_{i=1}^{I}[F(x_i)]^N \Omega(b_{i-1}|x_i)$$

Rearranging we get:

$$\textbf{(1)} \qquad P\sum_{i=1}^{I}\left\{[F(x_i)]^N - [F(x_i)]^N \Omega(b_{i-1}|x_i)\right\} = b_I\theta(b_I)$$

Equation (1) must hold in any pure-strategy equilibrium.

Substituting for $\Omega(b_i|\, x_{i-1})$ we can expand each term in the summation to be:

$$[F(x_i)]^N - [F(x_i)]^N \sum_{k=0}^{M-1}\binom{N}{k}\left(\frac{F(x_i)-F(x_{i-1})}{F(x_i)}\right)^k\left(\frac{F(x_{i-1})}{F(x_i)}\right)^{N-k}$$
$$- [F(x_i)]^N \sum_{k=M}^{N}\binom{N}{k}\left(\frac{F(x_i)-F(x_{i-1})}{F(x_i)}\right)^k\left(\frac{F(x_{i-1})}{F(x_i)}\right)^{N-k}\left(\frac{M}{k+1}\right)$$

This reduces to:

$$[F(x_i)]^N - \sum_{k=0}^{M-1}\binom{N}{k}(F(x_i)-F(x_{i-1}))^k(F(x_{i-1}))^{N-k}$$
$$- \sum_{k=M}^{N}\binom{N}{k}(F(x_i)-F(x_{i-1}))^k(F(x_{i-1}))^{N-k}\left(\frac{M}{k+1}\right)$$

From the *Binomial Theorem* we know: $\sum_{k=0}^{N}\binom{N}{k}(a)^k(b)^{N-k}=(a+b)^N$

Which implies:

$$\sum_{k=0}^{M-1}\binom{N}{k}\left(F(x_i)-F(x_{i-1})\right)^k\left(F(x_{i-1})\right)^{N-k}$$
$$=F(x_i)^N-\sum_{k=M}^{N}\binom{N}{k}\left(F(x_i)-F(x_{i-1})\right)^k\left(F(x_{i-1})\right)^{N-k}$$

Substituting into our expression, we know that each term in the summation equals:

$$\sum_{k=M}^{N}\binom{N}{k}\left(F(x_i)-F(x_{i-1})\right)^k\left(F(x_{i-1})\right)^{N-k}$$
$$-\sum_{k=M}^{N}\binom{N}{k}\left(F(x_i)-F(x_{i-1})\right)^k\left(F(x_{i-1})\right)^{N-k}\left(\frac{M}{k+1}\right)$$

Combining terms, each of our summation terms equals:

$$\sum_{k=M}^{N}\binom{N}{k}\left(F(x_i)-F(x_{i-1})\right)^k\left(F(x_{i-1})\right)^{N-k}\left(\frac{k+1-M}{k+1}\right)$$

Therefore, in any pure strategy equilibrium, the following must be true:

$$\textbf{(1')}\qquad P\sum_{i=1}^{I}\left\{\sum_{k=M}^{N}\binom{N}{k}\left(F(x_i)-F(x_{i-1})\right)^k\left(F(x_{i-1})\right)^{N-k}\left(\frac{k+1-M}{k+1}\right)\right\}=b_I\theta(b_I)$$

If the Ith critical value x_I is $\overline{x}$, this implies that $\overline{x}$ is the only firm that submits a bid of b_I therefore $\theta(b_I) = 1$. Thus we have the equation:

$$\textbf{(1'')}\qquad \frac{b_I}{P}=\sum_{i=1}^{I}\left\{\sum_{k=M}^{N}\binom{N}{k}\left(F(x_i)-F(x_{i-1})\right)^k\left(F(x_{i-1})\right)^{N-k}\left(\frac{k+1-M}{k+1}\right)\right\}$$

Expanding equation (1") we are left with the following for $\frac{b_I}{P}$:

$$\frac{b_I}{P} = \left\{\left(\frac{N+1-M}{N+1}\right)\left(F(x_1)\right)^N\right\} \qquad \text{(for } i = 1\text{)}$$

$$+ \left\{\begin{aligned} &\binom{N}{M}\left(\frac{1}{M+1}\right)\left(F(x_2)-F(x_1)\right)^M\left(F(x_1)\right)^{N-M} \\ &+\binom{N}{M+1}\left(\frac{2}{M+2}\right)\left(F(x_2)-F(x_1)\right)^{M+1}\left(F(x_1)\right)^{N-M-1} \\ &\qquad\vdots \\ &+\binom{N}{N-1}\left(\frac{N-M}{N}\right)\left(F(x_2)-F(x_1)\right)^{N-1}\left(F(x_1)\right) \\ &+\left(\frac{N+1-M}{N+1}\right)\left(F(x_2)-F(x_1)\right)^N \end{aligned}\right\} \qquad \text{(for } i = 2\text{)}$$

$$+$$

$$\vdots$$

$$+ \left\{\begin{aligned} &\binom{N}{M}\left(\frac{1}{M+1}\right)\left(1-F(x_{I-1})\right)^M\left(F(x_{I-1})\right)^{N-M} \\ &+\binom{N}{M+1}\left(\frac{2}{M+2}\right)\left(1-F(x_{I-1})\right)^{M+1}\left(F(x_{I-1})\right)^{N-M-1} \\ &\qquad\vdots \\ &+\binom{N}{N-1}\left(\frac{N-M}{N}\right)\left(1-F(x_{I-1})\right)^{N-1}\left(F(x_{I-1})\right) \\ &+\left(\frac{N+1-M}{N+1}\right)\left(1-F(x_{I-1})\right)^N \end{aligned}\right\} \qquad \text{(for } i = I\text{)}$$

Collecting terms:

$$\frac{\overline{b}_I}{P} = \left(\frac{N+1-M}{N+1}\right)\Big[(1-F(x_{I-1}))^N + (F(x_{I-1}) - F(x_{I-2}))^N + \ldots$$

$$\ldots + (F(x_2) - F(x_1))^N + (F(x_1))^N\Big]$$

$$+ \binom{N}{N-1}\left(\frac{N-M}{N}\right)\Big[(1-F(x_{I-1}))^{N-1}(F(x_{I-1})) + (F(x_{I-1}) - F(x_{I-2}))^{N-1}(F(x_{I-2})) + \ldots$$

$$\ldots + (F(x_2) - F(x_1))^{N-1}(F(x_1))\Big]$$

$$+ \binom{N}{N-2}\left(\frac{N-1-M}{N-1}\right)\Big[(1-F(x_{I-1}))^{N-2}(F(x_{I-1}))^2 + (F(x_{I-1}) - F(x_{I-2}))^{N-2}(F(x_{I-2}))^2 + \ldots$$

$$\ldots + (F(x_2) - F(x_1))^{N-2}(F(x_1))^2\Big]$$

$$+$$

$$\vdots$$

$$+ \binom{N}{M}\left(\frac{1}{M+1}\right)\Big[(1-F(x_{I-1}))^M (F(x_{I-1}))^{N-M} + (F(x_{I-1}) - F(x_{I-2}))^M (F(x_{I-2}))^{N-M} + \ldots$$

$$\ldots + (F(x_2) - F(x_1))^M (F(x_1))^{N-M}\Big]$$

However, all of the $\binom{N}{M}$ and $\left(\frac{N-M}{N}\right)$ terms are independent of *I*. Therefore, as *I* gets larger, we are simply dividing up the [0,1] interval into smaller and smaller *partitions*. There is no requirement that each partition is of equal size, but for any difference between F_i and F_{i-1}, we know that the largest possible value for that length is when it is composed of a single partition. For example, $\left(\frac{1}{2}\right)^2 > \left(\frac{a}{b}\right)^2 + \left(\frac{c}{d}\right)^2$ where *a/b* and *c/d* sum to 1/2 and *a,b,c,d* > 0. Therefore, even though the terms $(F_i - F_{i-1})$ are of potentially unequal sizes, if we can show that the "largest" of these partitions can be driven to zero, then we will have show

that *all* bids converge to zero as the bidding increment becomes arbitrarily small because the sum of all partitions cannot be larger than I times the largest partition.

To save notation, we'll label partitions, $(F_i - F_{i-1})$, as "a_i" for $i = 1,2,\ldots, I$.

By definition, $\sum_{i=1}^{I} a_i = 1$ and $N \geq 2$.

We define a_1 to be the weakly largest partition so that: $\forall i,\ a_1 \geq a_i$.

Lemma C2: $\frac{1}{a_1}(a_1)^N \geq \sum_{i=1}^{I} a_i^N$

Proof: $\forall i,\ a_1 \geq a_i \Rightarrow\ a_1^{N-1} \geq a_i^{N-1} \Rightarrow a_1^{N-1} \times a_i \geq a_i^N \Rightarrow a_i \geq \frac{a_i^N}{a_1^{N-1}}$.

Therefore: $\sum_{i=1}^{I} a_i \geq \sum_{i=1}^{I} \frac{a_i^N}{a_1^{N-1}}$ but we know by definition that $\sum_{i=1}^{I} a_i = 1$.

Therefore: $\frac{a_1}{a_1} \geq \sum_{i=1}^{I} \frac{a_i^N}{a_1^{N-1}}$

Multiply both sides through by a_1^{N-1} to get: $\frac{a_1^N}{a_1} \geq \sum_{i=1}^{I} a_i^N$

Rearrange to get: $\frac{1}{a_1}(a_1)^N \geq \sum_{i=1}^{I} a_i^N$

Q.E.D.

Therefore, since $\lim_{a \to 0} \frac{1}{a}(a)^N = 0$, if the largest bidding partition becomes arbitrarily small as the acceptable bidding increment converges to zero then we know that all bids also converge to zero.

As we make our bidding increments smaller and "add on" additional bids, we are simply adding on consecutive terms of $\left(F(x_j) - F(x_{j-1})\right)^m \left(F(x_{j-1})\right)^{n-m}$ and dividing up the [0,1] interval of *F(x)* into smaller and smaller units. Thus, we would logically believe that these intervals probably will converge to zero with small enough bidding increments.

Note the first partition *must* go to zero from the simple equilibrium equation:

$$\frac{b_1\theta(b_1)}{P} = \left\{\left(\frac{N+1-M}{N+1}\right)(F(x_1))^N\right\}$$

Clearly, as $b_1 \to 0$ then $F(x_1) \to 0$.

Looking at the next equation in line, we can also see that having "driven" $F(x_1) \to 0$, then we can also drive the partition $\left(F(x_1) - F(x_2)\right) \to 0$ because of the equilibrium equation:

$$\frac{b_2\theta(b_2)}{P} = \left\{\left(\frac{N+1-M}{N+1}\right)(F(x_1))^N\right\} + \left\{\begin{array}{l} \binom{N}{M}\left(\frac{1}{M+1}\right)(F(x_2)-F(x_1))^M (F(x_1))^{N-M} \\ + \binom{N}{M+1}\left(\frac{2}{M+2}\right)(F(x_2)-F(x_1))^{M+1}(F(x_1))^{N-M-1} \\ \qquad\vdots \\ + \binom{N}{N-1}\left(\frac{N-M}{N}\right)(F(x_2)-F(x_1))^{N-1}(F(x_1)) \\ + \left(\frac{N+1-M}{N+1}\right)(F(x_2)-F(x_1))^N \end{array}\right\}$$

So as we make $b_2 \to 0$, then the partition *(F_2 - F_1)* must also go to zero.

Obviously we can continue to do this for any *finite* number of intervals.

We can also show that we can always make the last partition go to zero. The equilibrium equation for the "last" partition is:

$$PF(x_I)^N\left[1-\Omega(b_{I-1}|x_i)\right] = b_I - b_{I-1}\theta(b_{I-1})$$

As $\Delta b \to 0$, this goes to:

$$PF(x_I)^N\left[1-\Omega(b_{I-1}|x_i)\right] = b_I\left[1-\theta(b_{I-1})\right]$$

Expanding this equation and rearranging we get:

$$(P-b_I)\sum_{k=M}^{N}\binom{N}{k}\left(1-F(x_{I-1})\right)^k\left(F(x_{I-1})\right)^{N-k}\left(\frac{k+1-M}{k+1}\right)=0$$

This implies that either $b_I \to P$ or $F(x_{I-1}) \to 1$.

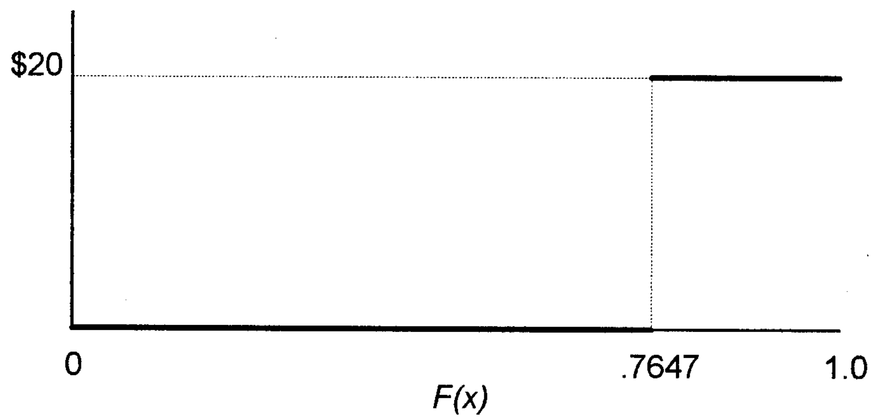
It cannot be true that $b_I \to P$ because this would mean the expected profits for the largest value of x are 0. Clearly the largest value of x expects positive profits by bidding an amount less than P since winning is guaranteed anytime entry is gained. Therefore, the only possible solution is for $F(x_{I-1}) \to 1$.

Therefore, I have shown that if we can make all partitions arbitrarily small by making the bidding increments small enough and "packing in" enough bids, then all bids must converge to zero. I have also shown that the first partition must converge to zero, along with any finite number of partitions beyond the first. Also I have shown that the last partition must converge to an arbitrarily small size. Unfortunately, I have not been able to prove that *all* partitions will become arbitrarily small because that involves solving for the equilibrium of a infinite number

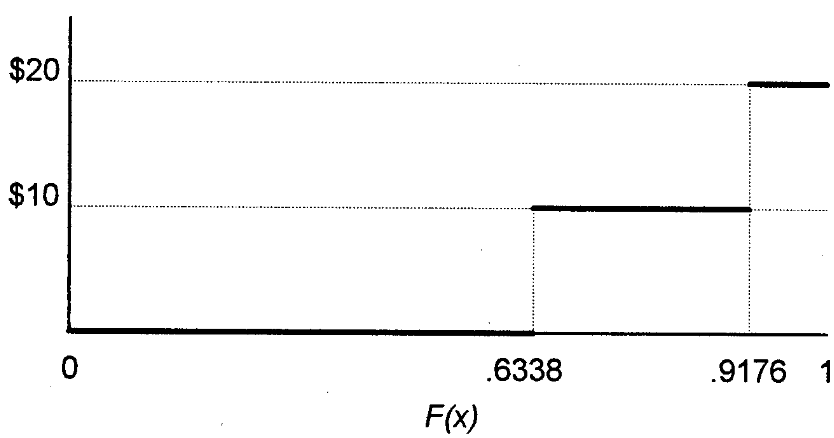
of simultaneous equations. Nevertheless, my analysis strongly suggests that this is what will occur. Therefore, I believe that as the sponsor reduces the acceptable bidding increment this will ultimately result in the sponsor receiving *less* entry fees.

To illustrate the response to differences in bidding increments, I have solved for the equilibrium in tournaments with six total firms from which the sponsor wants to select two entrants. I assume the sponsor is offering a $100 prize.

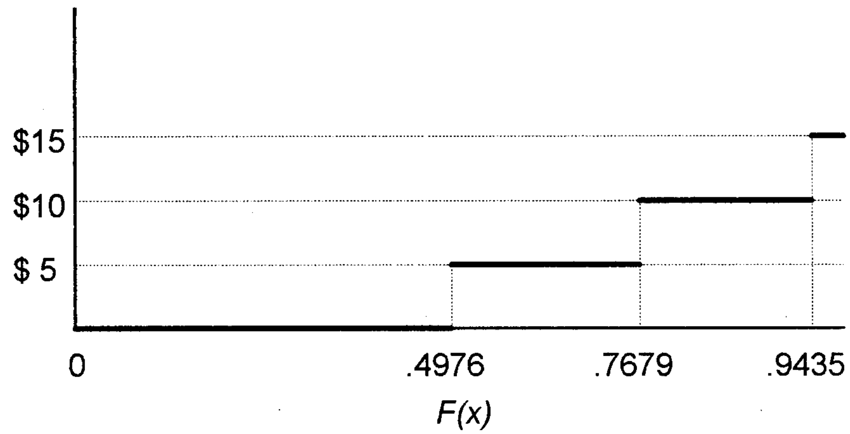
EXAMPLE C1: Bids only allowed in $20.00 increments.



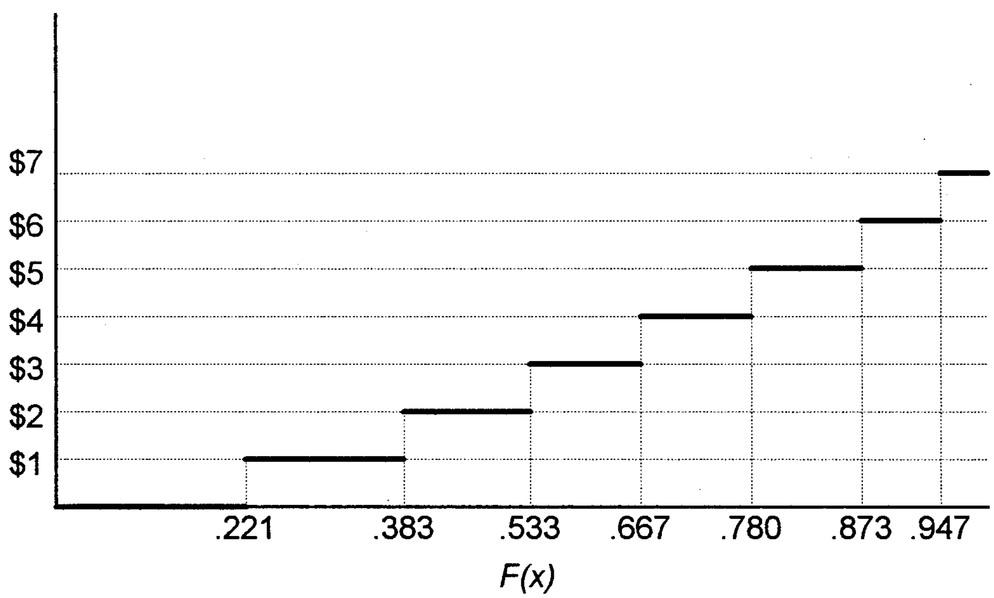
EXAMPLE 2: Bids only allowed in $10.00 increments.



EXAMPLE 3: Bids allowed only in increments of $5.00



Example 4: Bids allowed only in increments of $1.00



As can be seen from these examples, reducing the acceptable bidding increment generally lowers the highest bid made by the best firm. So, by reducing the

acceptable bidding increment, the sponsor is better able to distinguish between the quality of firms because the partitions are smaller, but at the same time the total entry fee collection will probably drop.

It is important to keep in mind that this auction cannot be efficient because it always requires significant probabilities of ties between bids by firms. Finally, if we take this to the limit and allow the bidding increments to converge to zero, the analysis suggests that all bids will also converge to zero. In chapter two using the theorem by Laffont and Guesnerie we saw that the only possible equilibrium in the case of continuous bids was for everyone to bid zero -- which obviously is not an equilibrium. Nevertheless, it corresponds to what seems to be happening in the cases above with discrete bids and therefore seems to confirm our analysis that bids will indeed converge to zero with small enough bidding increments.

Appendix 2A

Assorted Technical Details of Chapter 2

I. A direct proof of the optimal bidding function:

Firms want to maximize $KB(x) \times Pr[x\text{-}B(x) > y - B(y)]$

We're looking for a strictly increasing bidding function. So, let us define:

$H(y) := y - B(y)$. (and assume H has an inverse)

Now let us assume that x chooses to bid r.

For some y it is true that $[x - r] = y - B(y) = H(y)$. (else x always wins or loses)

Thus, $H^{-1}(x\text{-}r) = y$ and $\pi(r,x) = Kr[F\{H^{-1}(x\text{-}r)\}]^{M-1}$

Therefore, $\pi_r = K[F\{H^{-1}(x\text{-}r)\}]^{M-1} - Kr(M\text{-}1)\,[F\{H^{-1}(x\text{-}r)\}]^{M-2} f\{H^{-1}(x\text{-}r)\}H^{-1}{}'$

Now we set $r = B(x)$ and set $\pi_r = 0$ to get:

$K[F\{H^{-1}(x\text{-}B(x))\}]^{M-1} - KB(x)(M\text{-}1)\,[F\{H^{-1}(x\text{-}B(x))\}]^{M-2} f\{H^{-1}(x\text{-}B(x))\}H^{-1}{}' = 0$

Now in equilibrium, $H^{-1}(x\text{-}B(x)) = x$. So we substitute this in to get:

$$K[F(x)]^{M-1} = KB(x)(M\text{-}1)\,[F(x)]^{M-2} f(x)\, H^{-1}{}'$$

Cancel out the K's, and divide both sides by $[F(x)]^{M-2}$ to get:

$$F(x) = B(x)(M\text{-}1)\, f(x)\, H^{-1}{}'$$

But we know that $[H^{-1}\{x\text{-}B(x)\}] = x$. Therefore:

(*)
$$H^{-1}{}' = \frac{1}{1 - B'(x)}.$$

Substituting in for the above we get:

(**)
$$F(x)(1\text{-}B'(x)) = B(x)(M\text{-}1)\, f(x)$$

Rearranging:

$$B'(x) = 1 - \frac{B(x)(M-1)f(x)}{F(x)}$$

This is solved by $B(x_i) = \dfrac{\int_0^{x_i} [F(\xi_i)]^{M\text{-}1} d\xi}{[F(x_i)]^{M\text{-}1}}$, which is the bidding function.

Q.E.D.

NOTE if we did not use the impartial third party, then contestants would be maximizing: $\pi(r,x) = Kr[F\{H^{-1}(x-Kr)\}]^{M-1}$

In this case, equation (*) would then be: $H^{-1\prime} = \dfrac{1}{1 - KB'(x)}$.

and equation (**) would be: $F(x)(1-KB'(x)) = B(x)(M-1) f(x)$

Then this would leave us with the solution:

$$B'(x) = \frac{\left[1 - \dfrac{B(x)(M-1)f(x)}{F(x)}\right]}{K}$$

Which of course means that we have:

$$B(x_i) = \frac{\int_0^{x_i} [F(\xi_i)]^{M\text{-}1} d\xi}{\mathrm{K}[F(x_i)]^{M\text{-}1}}$$

Which identically nullifies our attempt at manipulating the *z-stop*.

<u>II. Proof that when $z = 0$, the expected cost of both types of tournaments is identical.</u>

In order to cut down on some of the extensive notation, in the rest of this appendix I define $G(\xi) = \left[[F(z)]^T + \left[1 - F(z)^T\right] \frac{F(\xi) - F(z)}{1 - F(z)} \right]$ which is the second-half of Φ.

The following equations implicitly define the *z-stop* values for the tournaments:

(1P) $$P \int_z^U \left[\Phi^{M-1}(y;z,T) - \Phi^{M-1}(z;z,T) \right] f(y)dy - C = 0$$

(1A) $$\int_z^U \left[B(y)\Phi^{M-1}(y;z,T) - B(z)\Phi^{M-1}(z;z,T) \right] f(y)dy - C = 0$$

We can also substitute Φ into the optimal bid function to see that the bids are:

$$B(x_i) = \frac{\int_0^{x_i} [\Phi(\xi_i)]^{M\text{-}1} d\xi}{[\Phi(x_i)]^{M\text{-}1}} = \begin{cases} \dfrac{\int_0^{x_i} [F^T(\xi_i)]^{M\text{-}1} d\xi}{[F^T(x_i)]^{M\text{-}1}} & x \leq z \\[2ex] \dfrac{\int_0^{z} [F^T(\xi_i)]^{M\text{-}1} d\xi + \int_z^{x} [G(\xi_i)]^{M\text{-}1} d\xi}{[G(x_i)]^{M\text{-}1}} & \text{for } x > z \end{cases}$$

Given *B(x)*, we can substitute into (1A) and through repeated substitution and cancellation one can show that (1A) simply reduces to the equation:

(1A') $$\int_z^U \int_z^y [G(\xi)]^{M-1} d\xi f(y)dy - C = 0$$

The expected cost of the "prize" in the auction-style tournament is just:

$$\text{(2A)} \qquad M\int_0^U B(x)\,\Phi(x{:}\,z,T)^{M-1}\,\phi(x{:}\,z,T)dx$$

The cost of the prize in a fixed-prize tournament is **P** and can be rewritten as:.

$$\text{(2P)} \qquad M\int_0^U P\Phi(x{:}\,z,T)^{M-1}\,\phi(x{:}\,z,T)dx$$

We want to prove that given the same *z-stop* , (2P) > (2A).

Since (1P) and (1A') implicitly define the *z-stops* for the two tournaments, the following equation must be true if both contests have the same *z-stop*:

$$\text{(3)} \quad P\int_z^U\left[\Phi^{M-1}(y;z,T)-\Phi^{M-1}(z;z,T)\right]f(y)dy-C=\int_z^U\int_z^y[G(\xi)]^{M-1}d\xi f(y)dy-C$$

On the left side: $\Phi^{M-1}(y,z,T)=[G(y)]^{M-1}$ and $\Phi^{M-1}(z,z,T)=\left[F^T(z)\right]^{M-1}$

so we can substitute in, rearrange, and cancel out the *C's* to get:

$$P\int_z^U\left[[G(y)]^{M-1}\right]f(y)dy=\int_z^U\left[\int_z^y[G(\xi)]^{M-1}d\xi+P\left[F^T(z)\right]^{M-1}\right]f(y)dy$$

z is fixed and identical for the tournaments, so we can multiply both sides by :

$\dfrac{1-F^T(z)}{1-F(z)}$ and when $y > z$ the density $\phi(y,z,T)=\dfrac{1-F^T(z)}{1-F(z)}f(y)$ giving us:

$$\text{(3')} \qquad P\int_z^U[G(y)]^{M-1}\phi(y)dy=\int_z^U\left[\int_z^y[G(\xi)]^{M-1}d\xi+P\left[F^T(z)\right]^{M-1}\right]\phi(y)dy$$

Expanding expression (2P):

$$M\int_0^z P\left[F^T(z)\right]^{M-1}\phi(y)dy + M\int_z^U P[G(y)]^{M-1}\phi(y)dy$$

Substituting (3') into the far-right term of (2P) gives us:

$$\textbf{(2P')}\quad M\int_0^z P\left[F^T(z)\right]^{M-1}\phi(y)dy + M\int_z^U\left[\int_z^y [G(\xi)]^{M-1}d\xi + P\left[F^T(z)\right]^{M-1}\right]\phi(y)dy$$

We want to compare (2P') with (2A) to determine which is larger and (2A) equals:

$$M\int_0^z B(y)\left[F^T(y)\right]^{M-1}\phi(y)dy + M\int_z^U B(y)\,[G(y)]^{M-1}\phi(y)dy$$

Substituting in for our known equilibrium bidding functions and canceling gives us:

$$\text{(2A')}\quad M\int_0^z\left[\int_0^y\left[F^T(\xi)\right]^{M-1}d\xi\right]\phi(y)dy + M\int_z^U\left[\int_0^z\left[F^T(\xi)\right]^{M-1}d\xi + \int_z^y[G(\xi)]^{M-1}d\xi\right]\phi(y)dy$$

Now, to compare (2A') with (2P') we can factor out the M's and cancel out the terms with $G(\xi)$ in both expressions to leave us with the following:

$$\text{(2P'')}\qquad P\int_0^z\left[F^T(z)\right]^{M-1}\phi(y)dy + P\int_z^U\left[F^T(z)\right]^{M-1}\phi(y)dy$$

$$\text{(2A'')}\qquad \int_0^z\left[\int_0^y\left[F^T(\xi)\right]^{M-1}d\xi\right]\phi(y)dy + \int_z^U\left[\int_0^z\left[F^T(\xi)\right]^{M-1}d\xi\right]\phi(y)dy$$

Now notice that if $z = 0$ the two above expressions are both equal! This proves that the expected cost of both tournaments is the same when $z = 0$. Q.E.D.

As additional notes: One can see from the above expression that if $z > 0$ we could just divide both sides by $[F^T(z)]^{M-1}$ (which we obviously cannot do if $z = 0$) and then the two expressions reduce to just:

(2*p*) P

(2*a*) $$\int_0^z B(y)\,\phi(y)dy + \int_z^U B(z)\phi(y)dy$$

One can easily see from the above expressions that using Lemma 1 (when $z > 0$) results in another proof of Theorem 1

III. Proof that if $z = 0$ exactly (so firms still have a pure-strategy of research option) then the expected profit of all firms in both tournaments is zero.

The expected profit for a firm in the auction-style tournament is:

$$\int_0^U B(x)\,\Phi(x{:}z,T)^{M-1}\phi(x{:}z,T)dx - C\frac{1-F(z)^T}{1-F(z)}$$

When $z = 0$, then $\frac{1-F(z)^T}{1-F(z)} = 1$ so this reduces to: $\int_0^U B(x)\,\Phi(x{:}z,T)^{M-1} f(x)dx - C$

But this is the same equation as our *z-stop* equation when $z = 0$, which of course equals 0. Therefore, when $z = 0$ each firm's expected profits also equal zero. The same proof works for the fixed-prize tournament. Q.E.D.

IV. Proof that Δ′(z) < 0 so there is a unique symmetric z-stop equilibrium.

Following Taylor, let us define the real valued function $\Delta(z)$ by:

$$\Delta(z) = \int_z^U \int_z^x [G(\xi)]^{M-1} f(x)\, d\xi\, dx \; - \; C$$

First it is shown that $\Delta'(z) < 0$, which implies that $\Delta(z) = 0$ at most once.

$$\Delta'(z) = \int_z^U \left[-[G(z)]^{M-1} + (M-1)\int_z^x [G(\xi)]^{M-2} \left\{ TF^{T-1}(z) - \frac{1-F^T(z)}{1-F(z)} \right\} \left(\frac{1-F(\xi)}{1-F(z)} \right) f(z) d\xi \right] f(x) dx$$

If we can show $\left\{ TF^{T-1}(z) - \frac{1-F^T(z)}{1-F(z)} \right\} \leq 0$, this is always negative and we are done.

When T = 1, then $\left\{ TF^{T-1}(z) - \frac{1-F^T(z)}{1-F(z)} \right\} = 0$. By induction, below I show that this term gets smaller at T gets larger. We want to determine the sign of:

$$\left\{ TF^{T-1}(z) - \frac{1-F^T(z)}{1-F(z)} \right\} \gtrless \left\{ (T+1)F^T(z) - \frac{1-F^{T+1}(z)}{1-F(z)} \right\}$$

Multiplying through by 1 - *F(z)* and subtracting 1 from each side we get:

$$TF^{T-1} - TF^T + F^T \quad \gtrless \quad (T+1)F^T - (T+1)F^{T+1} + F^{T+1}$$

Rearranging:

$$TF^{T-1}[1 - \mathrm{F}] \quad \gtrless \quad TF^{T-1}[1-F]F$$

Canceling we are left with: $1 > \mathrm{F}$

Which implies for all T > 1, this term is less than 0. So I have shown that $\Delta'(z) < 0$.

V. Derivation of equation (1A') by expanding the z-stop equation.

z is implicitly defined in the auction-style tournament by the equation:

$$\int_z^U \left\{ B(x)\, \Phi(x{:}\, z, T)^{M-1} - B(z)\, \Phi(z{:}\, z, T)^{M-1} \right\} f(x)dx - C = 0$$

We can substitute in for *B(x)* and *B(z)* to get:

$$\int_z^U \left\{ \left[\frac{\int_0^z \left[F^T(\xi)\right]^{M-1} d\xi + \int_z^x \left[G(\xi)\right]^{M-1} d\xi}{\left[G(x)\right]^{M-1}} \right] \left[G(x)\right]^{M-1} - \left[\frac{\int_0^z \left[F^T(\xi)\right]^{M-1} d\xi + \int_z^z \left[G(\xi)\right]^{M-1} d\xi}{\left[G(z)\right]^{M-1}} \right] \left[G(z)\right]^{M-1} \right\} f(x)dx - C = 0$$

But this equation easily reduces to:

$$\text{(1A')} \qquad \int_z^U \int_z^x \left[G(\xi)\right]^{M-1} f(x)\, d\xi\, dx - C = 0$$

VI. Solving for the equilibrium bids and *z*-stop in the auction-style tournament in the first example with M=2, T=2, C=1 and *x* is uniform [0,100].

$$\Phi(x;z,T)=\begin{cases}[F(x)]^T=[.01x]^2 & \text{if } x\le z\\ [F(z)]^T+\left[1-F^T(z)\right]\dfrac{F(x)-F(z)}{1-F(z)}=[.01z]^2+[1+(.01z)](.01x-.01z) & \text{for } x>z\end{cases}$$

$$\phi(x;z,T)=\begin{cases}T[F(x)]^{T-1}f(x)=.0002x & \text{if } x\le z\\ \dfrac{\left[1-F^T(z)\right]}{1-F(z)}f(x)=[1+.01z](.01) & x>z\end{cases}$$

$$B(x_i)=\frac{\int_0^{x_i}[\Phi(\xi_i)]^{M\text{-}1}d\xi}{[\Phi(x_i)]^{M\text{-}1}}=\begin{cases}\dfrac{\int_0^{x_i}[.01\xi]^2d\xi}{[.01x]^2}=0.33333x & \text{when } x\le z\\ \dfrac{\int_0^{z}[.01\xi]^2d\xi+\int_z^{x_i}[.01z]^2+[1+(.01z)](.01\xi-.01z)d\xi}{[.01z]^2+[1+(.01z)](.01x-.01z)} & \text{if } x>z\end{cases}$$

Simplifying:
$$B(x_i)=\begin{cases}0.33333x & \text{when } x\le z\\ \dfrac{\left[.005x^2-.01xz+.00005x^2z+.005z^2-.00001667z^3\right]}{.01x-.01z+.0001xz} & \text{if } x>z\end{cases}$$

Z-stop is defined by:

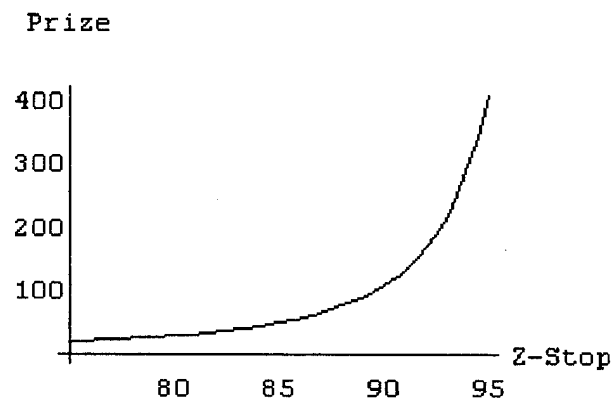
$$\int_z^U\left[KB(y)\Phi^{M-1}(y;z,T)-KB(z)\Phi^{M-1}(z;z,T)\right]f(y)dy-C=0$$

Substituting in our equations, letting K = 1 and simplifying we get:

$$\text{(1A)}\qquad \int_z^U\left[.00005x^2-.0001xz+.0000005x^2z+.00005z^2-.0000005z^3\right]dy-C=0$$

When C = 1, solving for the *z-stop* we get: *z = 84.2532*

VII. Below I plot the fixed-prize versus the equilibrium z-stop between z-stops of 75 and 95 in the first example with uniform[0,100], T=2, M=2, C=1, so the reader can see how the prize increases exponentially for larger z-stops. This is why it was so expensive to achieve a z-stop of 98.44 with the fixed prize but could be easily done in the auction.



Appendix 2B

Fixed-Prize and Auction-Style Tournament Examples

In this appendix I solve for the *z*-stops, expected best innovation, and expected net surplus for a number of tournaments to get a feel for the various trade-offs between fixed-prize tournaments and auction-style tournaments, and how manipulating the prize, number of entrants, and number of research periods will affect the net surplus of the sponsor. In all cases the distribution of innovations is uniform 0 to 100 and I always assume there are no costs for evaluating innovations.

<u>FIXED-PRIZE TOURNAMENTS</u>

<u>A. Fixed-Prize Tournament: T = 2</u>

In Table 2B.1 and Figure 2B.1, I give the *z-stops* in a fixed-prize tournament with T= 2. I vary the number of entrants, M, and the prize, P, for comparison.

Z-Stops: T = 2

M	P=5	P=7	P=8	P=9	P=10	P=12	P=15
2	48.0151	57.3939	60.5378	63.0865	65.2062	68.5548	72.1716
3	46.562	60.8907	64.8492	67.8429	70.208	73.7473	77.3396
4	24.8867	56.9417	63.478	67.7841	70.8948	75.1798	79.1788
5	0	39.6278	54.9537	63.6367	68.8023	74.8367	79.7192
6		16.6667	33.3211	49.3816	61.4247	72.6063	79.362
7		0	14.2857	28.5712	42.8251	66.1575	77.9792
8			0	12.5	25	49.9436	74.6576
9				0	11.1111	33.3333	65.818
10					0	20	49.9956
11						9.0909	36.3636
12						0	25
13							15.3846
14							7.1429
15							0

Table 2B.1

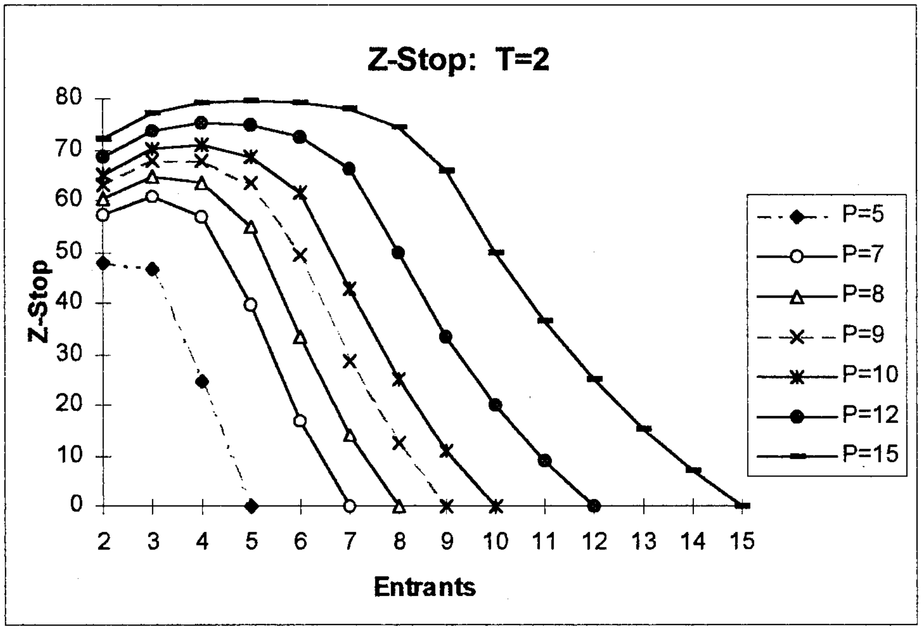


Figure 2B.1

Next, in Table 2B.2 and Figure 2B.2 I give the expected best innovation submitted to the sponsor in the fixed-prize tournament with two periods of research.

Expected Best Innovation Submitted to Sponsor: T=2

M	P=5	P=7	P=8	P=9	P=10	P=12	P=15
2	77.245	78.3332	78.6323	78.8509	79.0165	79.2485	79.4593
3	82.9123	84.3118	84.6199	84.8286	84.978	85.1754	85.3429
4	83.9855	87.2322	87.71	87.9883	88.1696	88.3892	88.5599
5	83.3333	88.0633	89.2397	89.7968	90.0889	90.3866	90.586
6		87.7551	89.2847	90.4365	91.1462	91.6973	91.9673
7		87.5	89.0625	90.2778	91.248	92.4763	92.9482
8			88.8889	90.1235	91.1111	92.5898	93.6305
9				90	91	92.5	93.9688
10					90.909	92.4242	93.9392
11						92.3611	93.8889
12						92.307	93.8462
13							93.8095
14							93.7778
15							93.75

Table 2B.2

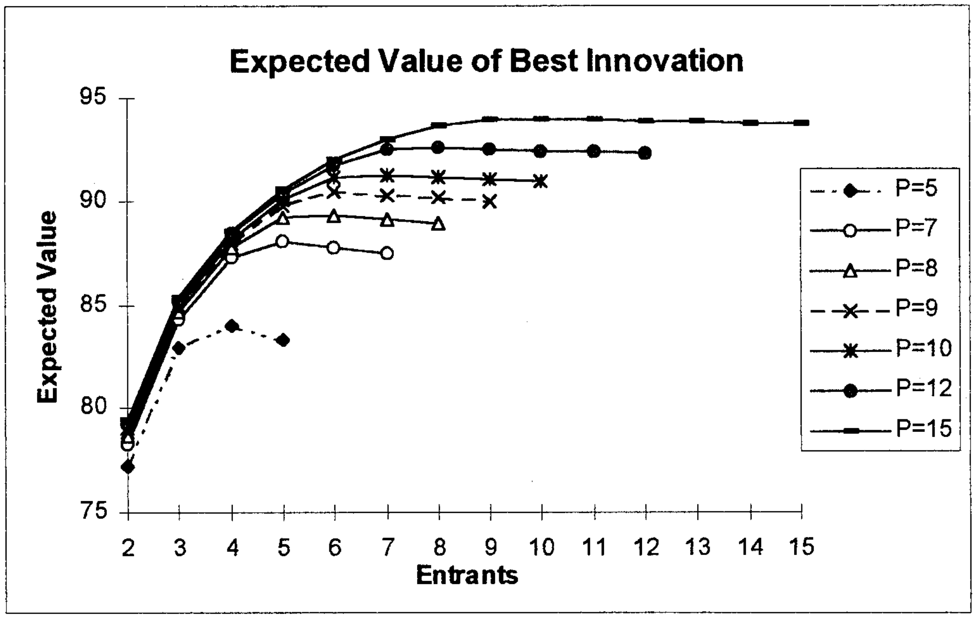


Figure 2B.2

Finally, in Table 2B.3 and Figure 2B.3 I plot the total expected net surplus in the fixed-prize tournament with two research periods. The expected net surplus is defined as the expected value of the best innovation minus the prize.

Expected Net Surplus: T=2

M	P=5	P=7	P=8	P=9	P=10	P=12	P=15
2	72.245	71.3332	70.6323	69.8509	69.0165	67.2485	64.4593
3	77.9123	77.3118	76.6199	75.8286	74.978	73.1754	70.3429
4	**78.9855**	80.2322	79.71	78.9883	78.1696	76.3892	73.5599
5	78.3333	**81.0633**	81.2397	80.7968	80.0889	78.3866	75.586
6		80.7551	**81.2847**	**81.4365**	81.1462	79.6973	76.9673
7		80.5	81.0625	81.2778	**81.248**	80.4763	77.9482
8			80.8889	81.1235	81.1111	**80.5898**	78.6305
9				81	81	80.5	**78.9688**
10					80.909	80.4242	78.9392
11						80.3611	78.8889
12						80.307	78.8462
13							78.8095
14							78.7778
15							78.75

Table 2B.3

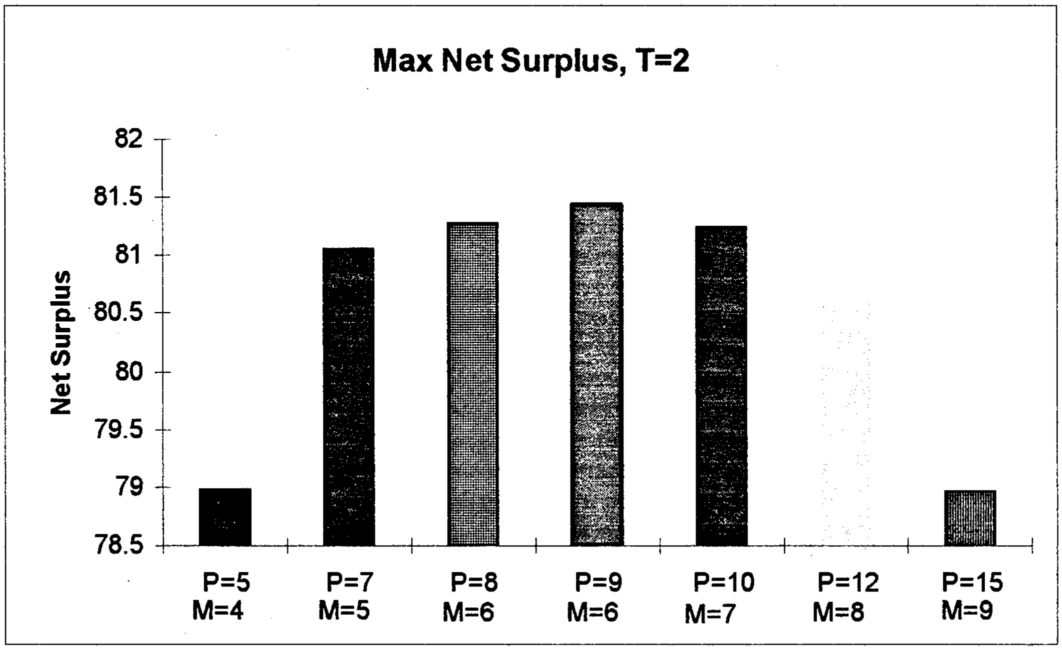


Figure 2B.3

In Figure 2B.3 above I charted the maximum expected net surplus for each prize when T = 2. As can be seen, <u>the optimal *fixed-prize* tournament when T = 2 is to offer a prize equal to 9 and to allow 6 entrants.</u> These values are based on the assumptions that there are no evaluation costs and the distribution of innovations is uniform 0 to 100. Obviously, if there are evaluation costs it is probable that you would be better off with fewer contestants and a smaller prize based on the information presented in Table 2B.3.

B. Fixed-Prize Tournament: T = 4

In Table 2B.4 and Figure 2B.4, I give the z-stops in a fixed-prize tournament with four periods of research. Again, I vary the number of entrants, M, and the size of the prize, P, for comparison.

Z-Stops: T = 4

M	P=5	P=8	P=9	P=10	P=11	P=12
2	55.7297	68.1329	70.4917	72.4145	74.0193	75.3839
3	41.6503	66.4893	70.3692	73.2643	75.5118	77.3135
4	20.1314	54.2068	61.517	67.2227	71.5058	74.706
5	0	38.9364	47.2004	54.3508	60.6286	66.0942
6		25.3077	34.2508	41.8384	48.4633	54.367
7		12.5215	22.4187	30.6149	37.6411	43.8169
8		0	11.1247	20.1314	27.701	34.2508
9			0	10.009	18.273	25.3077
10				0	9.0971	16.732
11					0	8.3377
12						0

Table 2B.4

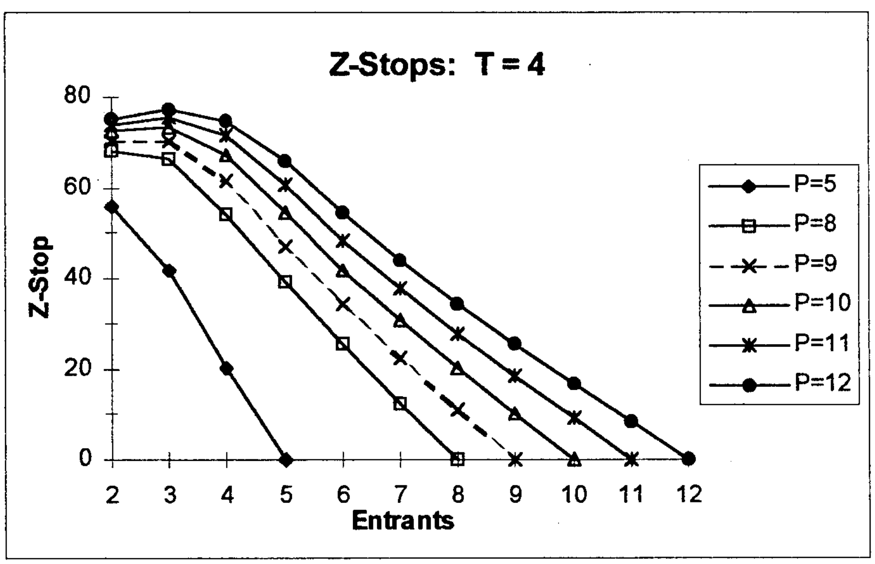


Figure 2B.4

Next, in Table 2B.5 and Figure 2B.5, I give the expected best innovation submitted to the sponsor in the fixed-prize tournament with four periods of research.

Expected Best Innovation: T = 4

M	P=5	P=8	P=9	P=10	P=11	P=12
2	83.6249	86.2438	86.658	86.9727	87.2187	87.4156
3	84.9598	89.5643	90.1415	90.5408	90.8303	91.048
4	84	89.9731	91.0159	91.7584	92.2737	92.6329
5	83.3333	89.5833	90.7405	91.6645	92.4129	93.0159
6		89.2857	90.4762	91.4286	92.2078	92.8569
7		89.0625	90.2778	91.25	92.0455	92.7083
8		88.8889	90.1235	91.1111	91.9192	92.5926
9			90	91	91.8182	92.5
10				90.9091	91.7355	92.4242
11					91.6667	92.3611
12						92.3077

Table 2B.5

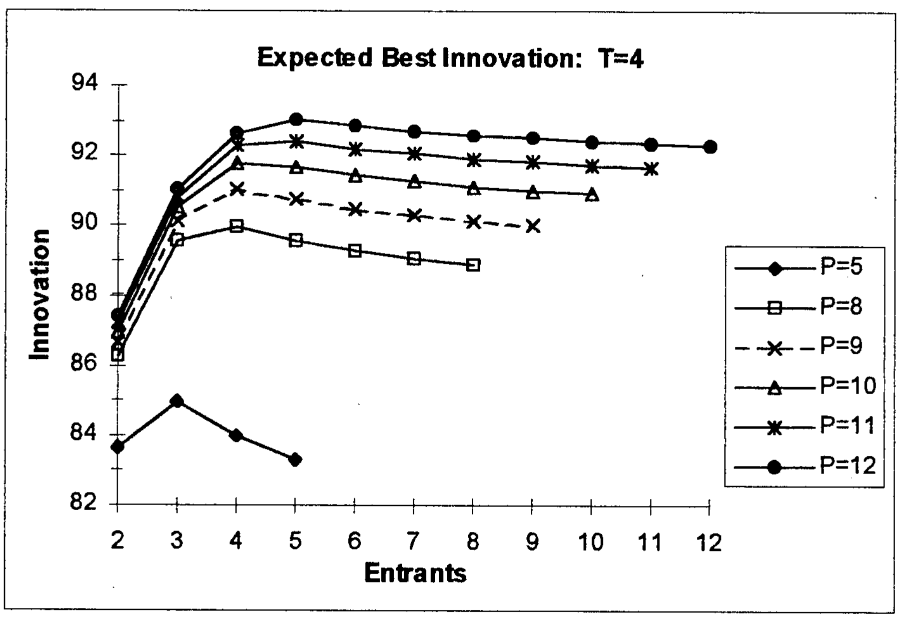


Figure 2B.5

Finally, in Table 2B.6 and Figure 2B.6 I give the expected net surplus in the fixed-prize tournament with four periods of research. Again, the expected net surplus is defined as the expected value of the best innovation minus the prize.

Expected Net Surplus: T = 4

M	P=5	P=8	P=9	P=10	P=11	P=12
2	78.6249	78.2438	77.658	76.9727	76.2187	75.4156
3	**79.96**	81.5643	81.1415	80.5408	79.8303	79.048
4	79	**81.973**	**82.016**	**81.758**	81.2737	80.6329
5	78.3333	81.5833	81.7405	81.6645	**81.413**	**81.016**
6		81.2857	81.4762	81.4286	81.2078	80.8569
7		81.0625	81.2778	81.25	81.0455	80.7083
8		80.8889	81.1235	81.1111	80.9192	80.5926
9			81	81	80.8182	80.5
10				80.9091	80.7355	80.4242
11					80.6667	80.3611
12						80.3077

Table 2B.6

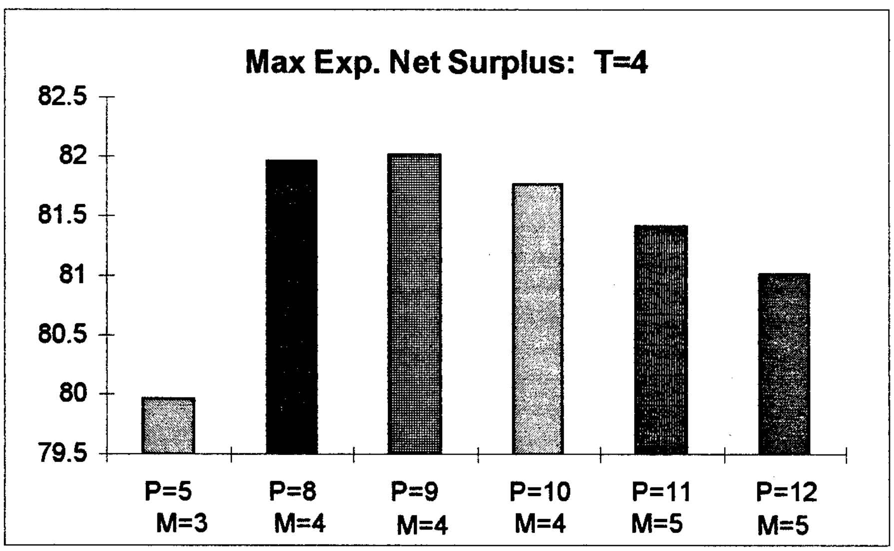


Figure 2B.6

In Figure 2B.6 above I charted the maximum expected net surplus for each prize when T = 4. As can be seen, <u>the optimal *fixed-prize* tournament when T = 4 is</u>

to offer a prize equal to 9 and allow 4 entrants to compete. Again, these values are based on a uniform 0 to 100 distribution of innovations and an assumption zero evaluation costs.

Comparing the T = 2 fixed-prize tournament to the T = 4 fixed-prize tournament we can see that raising the number of research periods did not change the optimal size of the prize but it did result in selecting fewer firms to compete. Also, the total net surplus to the sponsor increased when T was raised.

AUCTION-STYLE TOURNAMENTS

To analyze the auction-style tournament I again assume the distribution of innovations is uniform 0 to 100, and there are no evaluation costs. The tables below are separated according to the number of entrants, M, and I vary "K" as if I were using a third party to hold the bids. Thus K = 1 corresponds to an unaltered auction, while a value of K < 1 corresponds to an auction in which the third party holds the bids until after the sponsor has evaluated the innovations. With K < 1, the sponsor pays only a fraction of the bid submitted, as described in the text. Generally I selected K in multiples of 0.2, however in a few instances I have selected odd numbers for K which are in bold print. These values were selected because they generate *z-stops* which are identical to ones in previously listed fixed-prize tournaments so we can compare the net expected surplus. As can be seen, the auction-style tournament dominates the fixed-prize tournament for equal *z-stops*

which was proven in chapter two. However, since the *z-stops* are all relatively small, and I am comparing the auction-style tournaments with the optimum fixed-prize tournament the gains in net surplus are also relatively small.

C. Auction-Style Tournaments: T = 2

In all charts below, "Z" is the equilibrium z-stop, "E(x)" is the expected value of the best innovation submitted by all contestants, and "Net Surplus" is the expected surplus to the sponsor after receiving the innovation and paying *KB(x)*.

Auction-Style Results: M=2, T=2

K	Z	E(x)	Net Surplus
1	84.253	79.8945	53.3154
0.8	82.132	79.8414	58.6097
0.6	78.894	79.7472	63.8625
0.4	73.0661	79.5052	68.9739
0.2	57.5063	78.3444	73.1618

Table 2B.7

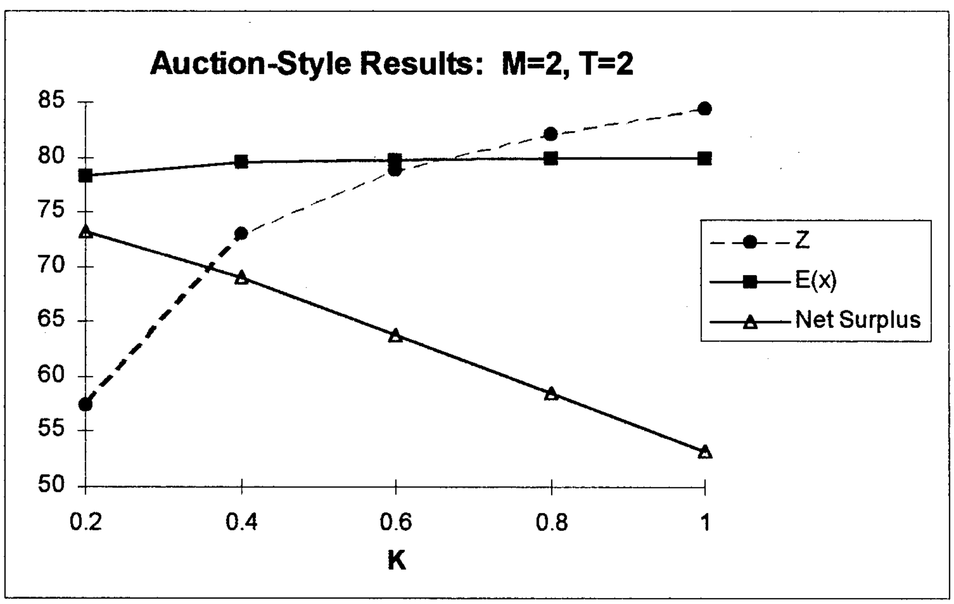


Figure 2B.7

On the previous page we can see that the net surplus substantially increases as K decreases with only two contestants and this distribution of innovations. This is because the two contestants make enough profit off of the contest that reducing K does not substantially reduce their equilibrium z-stop levels of effort and as can be seen it has relatively little effect on the expected best innovation, but it directly reduces the cost to the sponsor since the sponsor only pays *KB(x)*. Below I give the results when $M = 3$.

Auction-Style Results: M=3, T=2

K	Z	E(x)	Net Surplus
1	81.9919	85.5102	68.4933
0.8	79.0654	85.4114	71.8308
0.6	74.2379	85.2002	75.051
0.4	64.1069	84.5649	77.8059
0.26687	**46.562**	**82.9123**	**78.2193**
0.2	28.9567	80.6122	76.7205

Table 2B.8

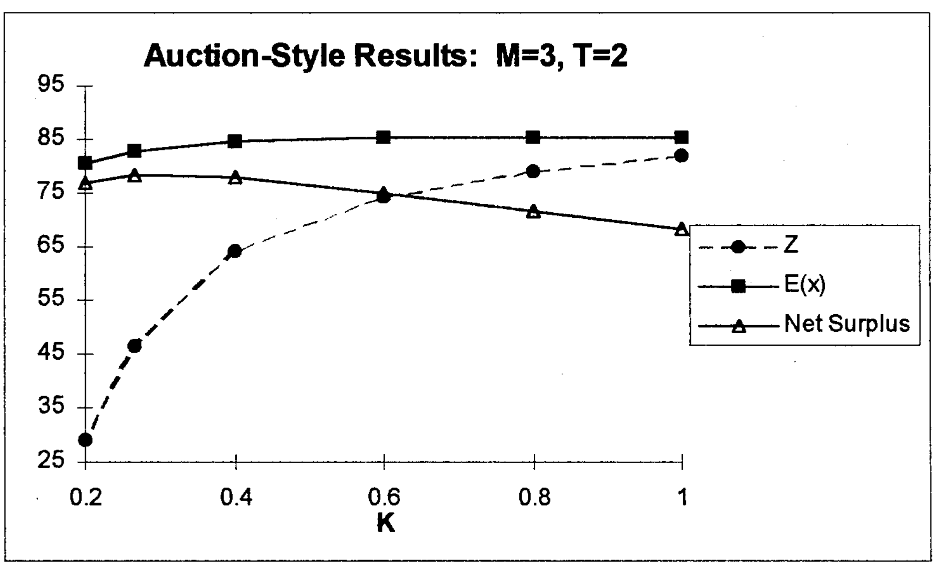


Figure 2B.8

Next, I raise the number of contestants to four firms.

Auction-Style Results: M=4, T=2

K	Z	E(x)	Net Surplus
1	78.4206	88.5302	75.9561
0.8	73.687	88.3168	78.2542
0.6	64.411	87.772	80.1283
0.51363	**56.9417**	**87.2322**	**80.5285**
0.4	41.169	85.8308	80.1443
0.2	0.00001	80	76

Table 2B.9

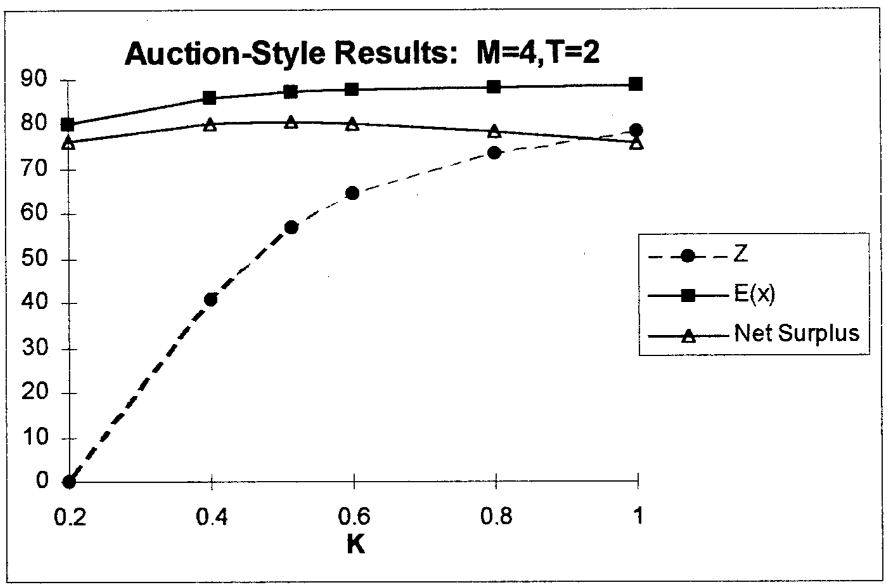


Figure 2B.9

With four contestants, the bidding competition at the end of the tournament is stiff enough that firms earn relatively small profits. This means that the z-stop is pretty large when K = 1, but that it falls off rapidly as we decrease K so that the gains in surplus from manipulating K are small, and below about K=0.5 the net surplus begins to fall as we continue to reduce K.

Auction-Style Results: M=5, T=2

K	Z	E(x)	Net Surplus
1	71.393	90.223	80.0928
0.8	60.9044	89.63	81.2278
0.72854	**54.9537**	**89.2397**	**81.3527**
0.6	41.367	88.2101	81.1315
0.4	15.47	85.5662	79.7927

Table 2B.10

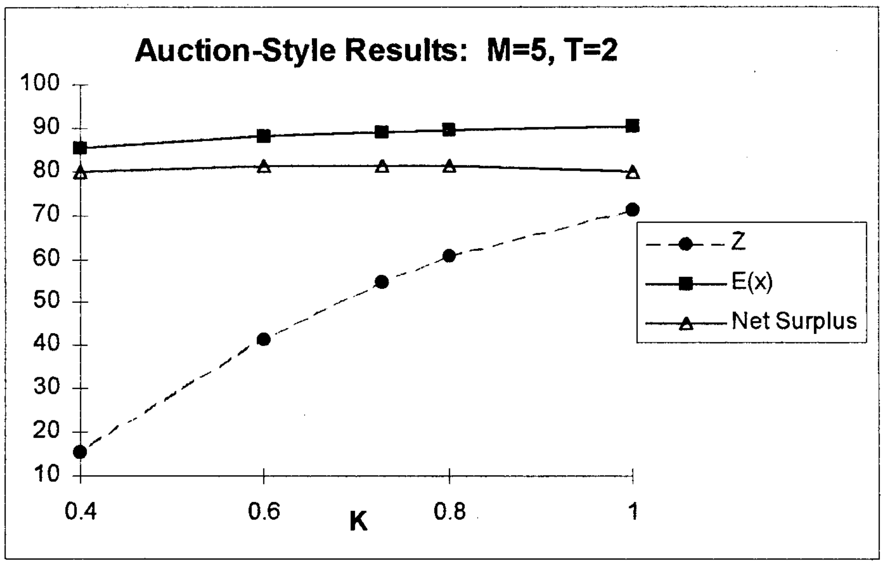


Figure 2B.10

Above the bidding competition with five contestants is so fierce and the z-stop falls so quickly that the gains from manipulating K are very small and the sponsor would probably be better off not even bothering with using a third party and just conducting a standard auction with K = 1.

Auction-Style Competition: M=6, T=2

K	Z	E(x)	Net Surplus
1	54.0515	90.7258	81.4243
0.9383	**49.384**	**90.4366**	**81.4567**
0.8	38.009	89.6484	81.3672
0.6	19.5227	88.0477	80.8763

Table 2B.11

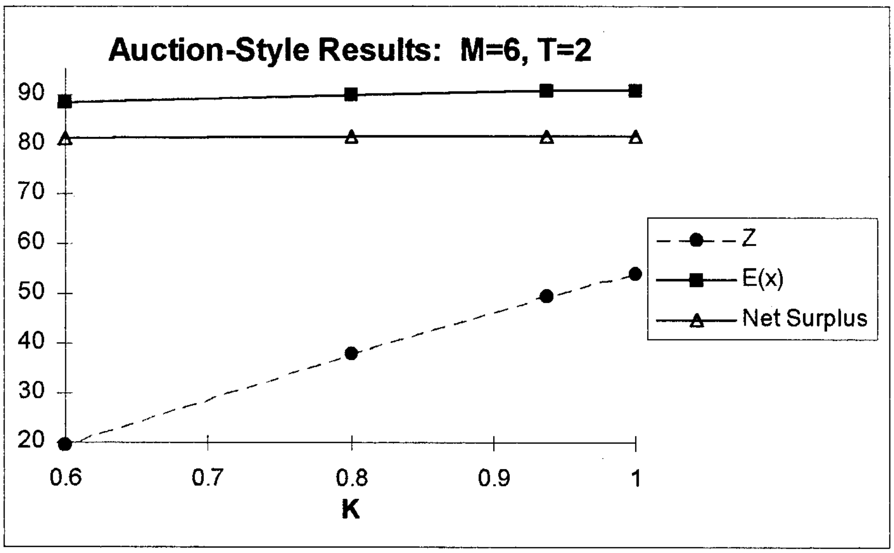


Figure 2B.11

Again, when there are six contestants the value of including a third party to hold the bids so the sponsor can manipulate K is very small. Essentially, the bidding competition is so great that there are not enough "profits" to extract out of contestants through using K to make it worthwhile.

D. Auction-Style Tournaments: T = 4

Below I give the same statistics as in the previous graphs, but now I allow contestants the opportunity to conduct research in four periods instead of just two.

Auction-Style Results: M=2, T=4

K	Z	E(x)	Net Surplus
1	82.3139	88.2281	70.8508
0.8	79.6054	87.9491	74.1332
0.6	75.3394	87.4094	77.1294
0.4	67.4211	86.113	79.2695
0.25664	**55.7297**	**83.6249**	**79.0206**
0.2	47.9022	81.6568	77.8241

Table 2B.12

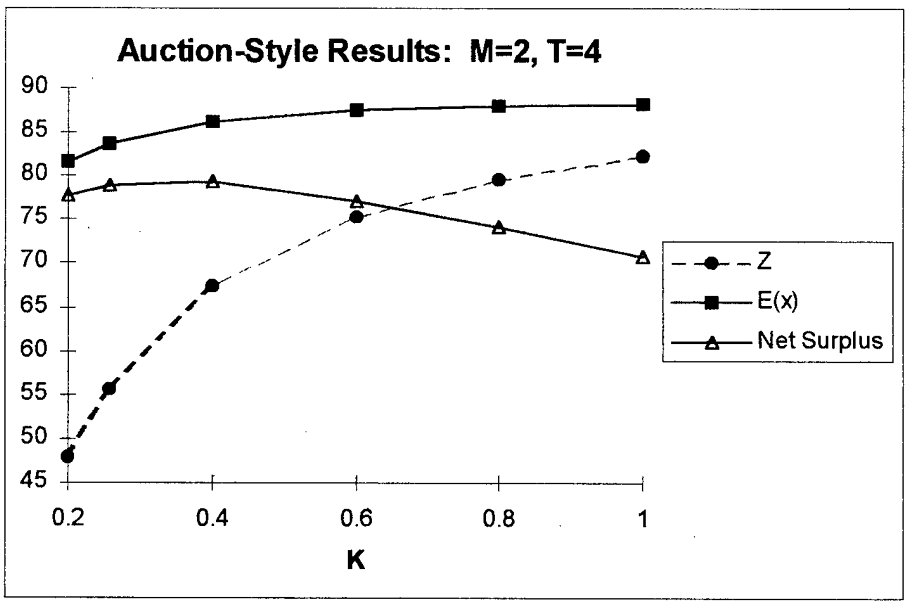


Figure 2B.12

Once again with only two contestants the net surplus is initially increasing as K is decreased. However, with T = 4 the effect of a lower z-stop is more pronounced and profits at the high end are smaller, so the increase in surplus is smaller for a

given reduction in K. By the time K gets to about 0.4, surplus begins to decrease with smaller K which did not happen when T = 2 and only two firms.

Auction-Style Results: M=3, T=4

K	Z	E(x)	Net Surplus
1	75.1479	90.7847	80.5778
0.8	69.7855	90.0576	81.6085
0.70982	**66.4893**	**89.5643**	**81.8419**
0.6	61.4585	88.7521	81.935
0.4	48.1582	86.3024	80.8089
0.2	22.7476	80.635	76.762

Table 2B.13

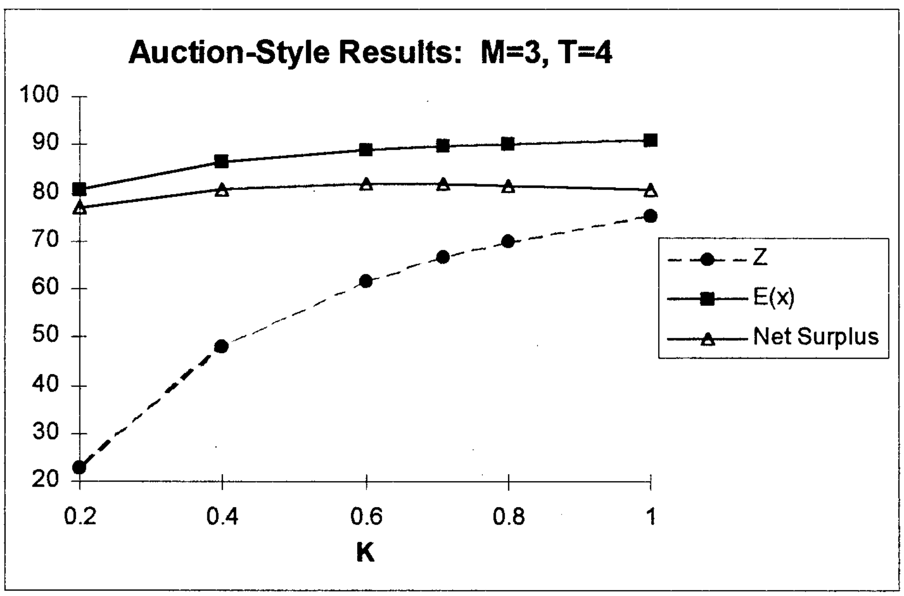


Figure 2B.13

Auction-Style Results: M=4, T=4

K	Z	E(x)	Net Surplus
1	61.7382	91.0559	82.0586
0.99324	**61.517**	**91.0159**	**82.061**
0.8	54.3597	89.9985	81.9913
0.6	44.536	88.4529	81.5242
0.4	29.8506	85.8578	80.201

Table 2B.14

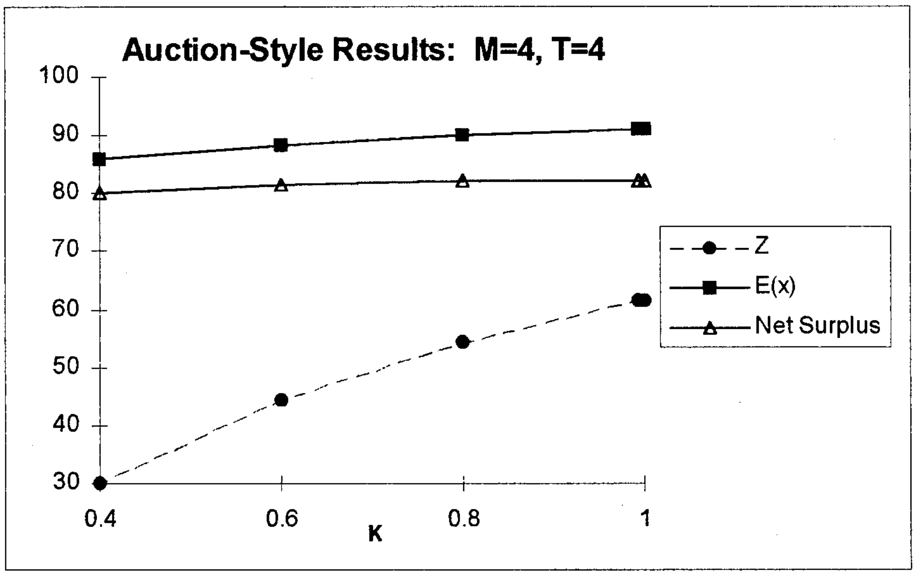


Figure 2B.14

Notice that with four contestants and T = 4 the net expected surplus is almost at its maximum in the unaltered auction with K = 1 and the unaltered auction performs better than all of the fixed-prize tournaments we analyzed. Notice that as we raise the number of periods of research that the optimal number of firms decreases in the auction-style tournament also. With only two research periods it was best to have six firms but with four periods of research it is best to have only four firms.

Auction-Style Results: M=5, T=4

K	Z	E(x)	Net Surplus
1	48.1787	90.8713	81.7424
0.9	44.5364	90.3775	81.7172
0.8	40.393	89.7938	81.6288

Table 2B.15

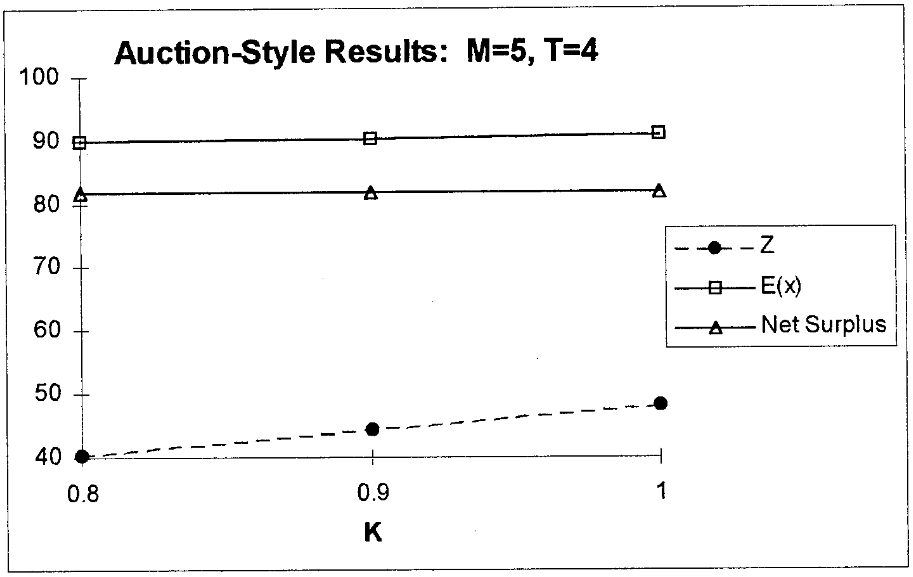


Figure 2B.15

I have included the numbers for M = 5 only down to K = 0.8 because I only wanted to show that net surplus is decreasing in K and the sponsor does better with four firms than with five. Thus, as the number of periods of research increases the optimal number of firms in the auction-style tournament decreases.

Finally, on the following page, I have plotted the associated results for both tournaments when M = 3 on the same figure so that it will be easier to make a comparison between the two tournaments. In all of the auction-style tournaments below, I have just left K = 1 as if there were no third party.

M = 3

T	Z	E[x]	E[x-B(x)]	*E[x-P]*	E[B(x)]	*Req. Fixed-P*
2	81.9919	85.5102	68.4933	63.8293	17.0169	21.6809
3	78.9653	89.3066	76.69	75.1505	12.6166	14.1561
4	75.1479	90.7847	80.5778	79.9632	10.2069	10.8215
5	71.5176	91.222	82.0649	81.7932	9.1571	9.4288
6	69.0372	91.3161	82.5031	82.3846	8.813	8.9315
7	67.5704	91.3347	82.6254	82.5748	8.7093	8.7599
8	66.7145	91.3386	82.6617	82.6402	8.6769	8.6984

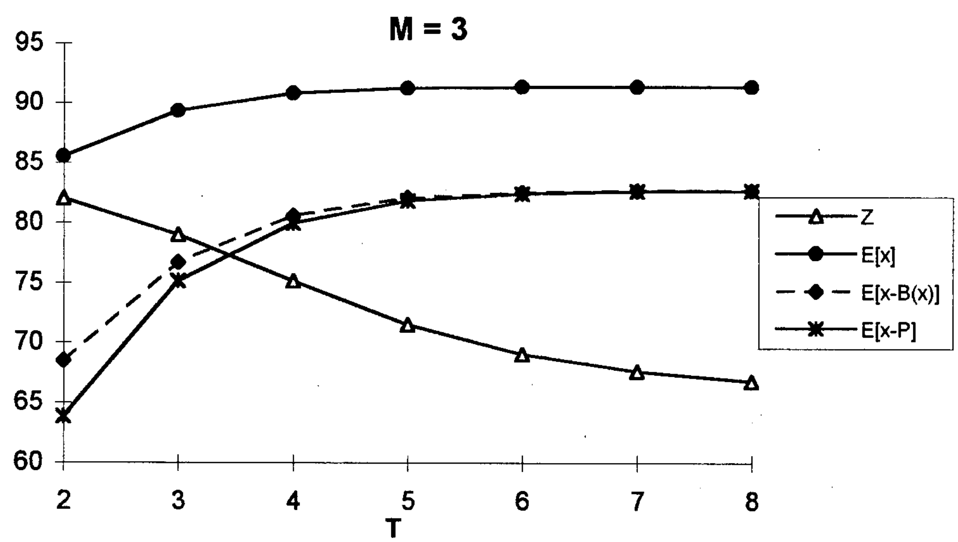


Figure 2B.16

Obviously from the figure above we can see that as the length of the periods grows, the difference between the expected net surplus of the fixed-prize tournament and the auction-style tournament shrinks, although as predicted the auction-style tournament always has a lower expected cost and higher expected surplus.

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